ON THE DISTRIBUTION OF COMPLETION TIMES FOR RANDOM COMMUNICATION IN THE TASK-ORIENTED GROUP WITH A SPECIAL STRUCTURE

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§ 1. Introduction.

Recently, A. Bavelas, L. S. Christie, R. D. Luce and J. Macy, Jr. have introduced the task-oriented group. According to them, the task-oriented group consists of a number of individuals and a communication network. And each individual has initially one piece of information which must be transmitted to all the others to complete the task. At every sending time each individual sends all the information he has required to one other individual chosen at random from the possibilities given by the communication network.

By introducing a Markov chain H. G. Landau [1] has shown how to calculate the distribution of the completion times. But this method needs the transition probabilities $a_{\alpha\beta}$ from the information state $c^{(\alpha)}$ to $c^{(\beta)}$ after one sending time. And it seems to be generally difficult to calculate $a_{\alpha\beta}$.

Now we shall denote by $T(l, m, n)$ the task-oriented group with the network indicated by Figure 1 where the numbers of links in $\overrightarrow{ecd}$, $\overrightarrow{dbe}$ and $\overrightarrow{dae}$ are respectively l, m and n. $T(l, m, n)$ is the simplest case from a topological view-point, because it is shown by R. D. Luce's theorem [2] that $T(l, m, n)$ is of order 1 free from tree form. In our paper we shall give the distributions of completion times of $T(l, m, n)$ for the following exclusive cases.

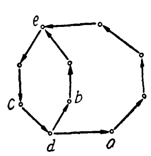

Figure 1.

Case (I): $m = n \geqq 2$;

Case (II): $m = 1,\ n \geqq 2$;

Case (III): $m + 1 = n,\ m \geqq 2$;

Case (IV): $m + 1 < n \leqq 2m,\ m \geqq 2$;

Case (V): $2m < n,\ m \geqq 2$.

We can assume $m \leqq n$ without loss of generality owing to the symmetricity property on m and n. And the discussion for case $m = n = 1$ is trivial. Hence we shall omit the case $m > n$ and $m = n = 1$ in this paper. All cases except these two are included in the above five cases.

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