ON THE APPROXIMATION TO SOME LIMITING DISTRIBUTIONS

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It is well known that

a) binomlal distribution with mean np, variance npq approaches to Poisson distribution with $\lambda = np$ as mean and variance when $n \to \infty$,

b) let X_i $(i = 1, 2, \cdots, n)$ obey the uniform distribution :

$$\begin{aligned} P(X_i \leqq x) &= 1 & x &\geqq \frac{1}{2}, \\ &= x + \frac{1}{2} & |x| &\leqq \frac{1}{2}, \\ &= 0 & x &\leqq -\frac{1}{2}, \end{aligned}$$

then $\sum_{i=1}^{n} X_i/\sqrt{n/12} \to N(0, 1)$ as $n \to \infty$.

With respect to these facts, we shall deduce the approximation formulas for the above two distributions, and also evaluate the absolute error.

Concerning these problems,

a) the relative error in the Poisson approximation to binomial distribution was estimated by W. Feller [3, p. 114] and J. Uspensky [4, p. 137],

b) the distribution of the sum of uniformly distributed random variables was discussed by Uno [5] and Uspensky [4]. In [5] the different method to compute the exact value of this distribution and the table (for $n \leqq 10$) are given.

Our approximation formula is an improvement of Uspensky's. Same methods as in [1], [2] can also be used for our purposes, so we omit the detail of them here.

§ 1. Poisson approximation to the binomial distribution.

Let X_i be as follows:

$$P(X_i = 1) = p,$$
$$P(X_i = 0) = q,$$

then $\sum_{i=1}^{n} X_i$ is a random variable which obeys the binomial distribution with mean np, variance npq and has a c. f.:

$$f_n(t) = (q + pe^{it})^n = \{1 + p(e^{it} - 1)\}^n, \tag{1}$$

and corresponding Poisson distribution with mean, variance $\lambda = np$ has

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