PROBABILISTIC INVESTIGATIONS ON POPULATION GENETICS

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SUMMARY

In a series of successive papers we have developed a unified theory of inheritance from a probabilistic view-point. They have been mainly concerned with the distributions in an ordinary population. In succession, we now deal with analogous problems in a population which consists of families involving several types of consanguineous marriages.

We consider again a single inherited character which consists of any number of multiple alleles at one diploid locus and of which the inheritance is subject to Mendelian law. Our main purpose is then to study the distributions of genotypes in several definite combinations, lineal or collateral, which consist of individuals in families possibly involving consanguineous marriages chosen at random from a population under consideration.

By making use of relative frequencies of genes as basic quantities, we derive systematically the formulas expressing probabilities of several combinations. It is well clarified how the correlations between genotypes of family-members are affected by intervention of consanguineous marriages. On the other hand, many notable phenomena are elucidated. Especially, the distribution of genotypes in a generation immediate after any consanguineous marriage deviates from an equilibrium state in such a manner that the frequency of any homozygous type increases while that of any heterozygous one decreases. This fact will illustrate a cause of a deviation observed on a distribution in an isolated population.

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Introduction. In a series of successive papers[1] we have developed a unified theory of inheritance from a probabilistic view-point. It has been the purpose of these papers to investigate the phenomenon of inheritance based upon a general mode which is supposed to be subject to Mendelian law and then to apply the results to some practical problems on forensic medicine. One of the main problems has been concerned in determining the distributions of genotypes in several definite combinations of individuals chosen at random from a population of a given equilibrium state. The mother-child combination, among others, has played a distinguished role. This concept is, as its own name shows, a combination consisting of an individual and its immediate descendant. It will be plausible, for instance, to attempt to generalize the concept to a lineal combination consisting of an individual and its distant descendant after any definite number of generations.

The main purpose of the present report[2] is to discuss further problems of determining the distributions of genotypes in several definite combinations, lineal or collateral, consisting of members in a family possibly involving consanguineous marriages. A population in consideration is again supposed to be in an equilibrium state unless the contrary is stated. It is further supposed that panmixia takes place at any generation except when consanguineous marriages are appointed.

Now we consider again a single inherited character which consists of m multiple alleles at one diploid locus denoted by A_i $(i=1,\cdots,m)$ and