ALTERNATIVE EXPRESSIONS FOR PROBABILITY-GENERATING FUNCTIONS CONCERNING AN INHERITED CHARACTER AFTER A PANMIXIA

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1. Introduction.

In a recent paper[1] we have discussed from a stochastic view-point a problem of estimating the distributions concerning an inherited character which consists of m multiple alleles at one diploid locus denoted by

$$A_i \qquad (i=1,\cdots,m)$$

and of which the inheritance is subject to Mendelian law. In succession, we have also discussed a related problem on mother-child combinations.[2),3)] Main tasks of these papers have been to obtain the expressions for respective probability-generating functions in explicit manners.

In any case, the generating function must be, of course, uniquely determined in a definite manner. However, our results have concerned not directly the generating functions themselves but somewhat indirectly the related functions from which the generating functions can be obtained as the constant terms of respective Laurent expansions with respect to parameters involved. Under such circumstances, it will be possible to find alternative sources of generating functions. In the present paper we shall illustrate the circumstances by deriving some alternative expressions for probability-generating functions.

As in the preceding papers, we consider a population of size $2N$ consisting of N females and N males. Let the given distributions of genotypes $\{A_a A_b\}$ in females and in males be designated by

$$\mathcal{F}=\{F_{ab}\} \quad \begin{pmatrix} a,b=1,\cdots,m; \\ a\leqq b \end{pmatrix}$$

and

$$\mathcal{M}=\{M_{ab}\} \quad \begin{pmatrix} a,b=1,\cdots,m; \\ a\leqq b \end{pmatrix},$$

respectively, so that

$$\sum_{a\leqq b} F_{ab}=\sum_{a\leqq b} M_{ab}=N.$$

The order of genes in a genotype being immaterial, any quantity accompanied by a pair of suffices indicating genes of a genotype should be supposed to be understood as symmetric with respect to the suffices; for instance, we suppose $F_{ab}=F_{ba}$, etc.

2. Mother-child combinations with mothers of an assigned genotype.

We first consider the mother-child combinations with mothers of an assigned genotype, $A_\alpha A_\beta$ say. Introducing a set of $m(m+1)/2$ stochastic variables

$$\mathcal{X}=\{X_{fg}\} \quad \begin{pmatrix} f,g=1,\cdots,m; \\ f\leqq g \end{pmatrix},$$

we designate, in conformity with a notation used in a previous paper[3)], by

$$\Psi(\alpha\beta|\mathcal{X})\equiv\Psi(\alpha\beta|\mathcal{X}|\mathcal{F};\mathcal{M})$$

the probability that, after a panmixia, the mother-child combinations $(A_\alpha A_\beta; A_f A_g)$ $(f,g=1,\cdots,m;\ f\leqq g)$ amount to X_{fg}, respectively. Here, and also in the subsequent discussions, each mating is supposed to produce one child so that in our present case there holds

$$\sum_{f\leqq g} X_{fg}=F_{\alpha\beta}.$$

Since a mother of any type $A_\alpha A_\beta$ can produce, in general, merely a child of a type involving at least a gene in common with herself, the probability $\Psi(\alpha\beta|\mathcal{X})$ must vanish out unless the X's are equal to zero for all the impossible children' types $A_f A_g$, i. e. for $f, g\neq\alpha, \beta$. The probability-generating function is now defined by