

TWO REMARKS ON MY PAPER

"A NOTE ON THE MAXIMAL IDEALS OF ANALYTIC FUNCTIONS"

By Sin HITOTUMATU

0. After publication of my previous paper mentioned in the title¹⁾, Prof. Henri CARTAN was so kindly to tell me the following two remarks. The author expresses his hearty gratitude to Prof. H. CARTAN for his kind advices. Also the author thanks Mr. Jun-ichi IGUSA for his valuable comments on the latter point.

1. It was very simple that not every closed ideal has finite basis. An example in the whole space of complex two variables x and y is obtained as the closure-ideal of the one generated by the following functions

$$f_0(x), \quad y f_1(x), \quad y^2 f_2(x), \\ \dots, \quad y^i f_i(x), \quad \dots,$$

where $f_i(x)$ are entire functions of a complex variable x with zero-points of $(j-i)$ -th order at $x=j$ ($j=i+1, i+2, \dots$).

2. The latter remark is less simple. The footnote 6) must be deleted, for the condition "with at most countable basis" seems to be essential! In fact, it has been shown by Mr. J. IGUSA²⁾ that if an ideal \mathcal{I} has the property that $\mathcal{I} \neq \mathcal{O}$ and $\overline{\mathcal{I}} = \mathcal{O}$, using ZORN's Lemma, there exists a maximal ideal $\mathcal{J} \neq \mathcal{O}$

containing \mathcal{I} , which satisfies a fortiori $\overline{\mathcal{J}} = \mathcal{O} \neq \mathcal{J}$. Therefore our Theorem 2, saying that a maximal ideal with at most countable basis is closed, is no longer true for general ideals. But by above Theorem 2 guarantees that such non-closed maximal ideal as above \mathcal{J} , cannot have finite or countable basis, and hence we have also an example of ideals without at most countable basis.

Added in proof: The author expresses also his gratitude to Prof. Melvin HENRIKSEN, who has told me the latter remark after completion of this manuscript.

- 1) S. Hitotumatu, A note on the maximal ideals of analytic functions, Kōdai Math. Sem. Reports 4, No.2 (1952), 51-53.
- 2) J. Igusa, On a property of the domain of regularity, Memoirs Univ. Kyoto 27, No.2 (1952), 95-97.

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