ON DISCRETE HOMOGENEOUS CHAINS

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Introduction. The object of the present paper is to determine some types of discrete homogeneous chains.

Because all discrete homogeneous chains correspond one-to-one to all homogeneous chains, as will be shown later, it is impossible to classify and determine the types of all descrete homogeneous chains, unless the types of all homogeneous chains are determined. So the author only determined the special type of discrete homogeneous chains, that is, absolutely discrete homogeneous chains, which will be defined later on.

The same definitions and notations as in [1] are employed, but concerning the ordinal power, those in the author's paper [2] are employed.

In §1, general homogeneous chains are investigated.

In §2, the construction of general discrete homogeneous chains is studied, and later the absolute discreteness is defined.

In §3, some examples of absolutely discrete homogeneous chains are investigated.

In §4, we shall see that every absolutely discrete homogeneous chain is after all one of examples mentioned in §3, and then the type is determined.

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1. On homogeneous chains.

(1.1) We use the same definitions and notations as in [1], unless otherwise mentioned, but concerning the definition of ordinal power, we use the following one, on which the author has studied in a previous paper [2].

Definition I. Let X and Y be posets, and y_0 be a fixed element of Y. The ordinal power ${}^X Y\langle y_0\rangle$ consists of all functions $f(x)=y$ from X to Y, such that 'the set $\{x \mid f(x) \neq y_0\}$ satisfies the descending chain condition', where $f \leqslant g$ means that for each $x \in X$ such that $f(x) \nleqslant g(x)$, there exists an $x' < x$ such that $f(x') < g(x')$.

Based on this definition, we get a poset ${}^X Y\langle y_0\rangle$ without any restriction on the original posets, such as the descending chain condition on X.

If Y is homogeneous, then the structure of ${}^X Y\langle y_0\rangle$ does not depend on the choice of y_0, and the sign $\langle y_0\rangle$ can be omitted. In the case when Y is homogeneous, the resultant set ${}^X Y$ is also homogeneous. In the case when both X and Y are chains, the set ${}^X Y\langle y_0\rangle$ is also a chain.

About those fact, see the previous paper [2].

(1.2) Definition 2. If a chain X has a transitive automorphism group, we call X homogeneous.

Theorem 1. If X and Y are homogeneous chains, then $X \circ Y$ (ordinal product, cf. [1], p.9) is a homogeneous chain. If X is a chain, and Y is a homogeneous chain, then ${}^X Y$ is a homogeneous chain.

These propositions are corellaries of Theorem II and III of [2].

Note I. Let $Z = X \circ Y$. The homogeneity of X and Y implies that of Z. But neither the homogeneity of Z and X nor that of Z and Y implies that of the rest. Z may be homogeneous when neither X nor Y is homogeneous. If Z and