where on the right side $x^j = f_1^j(t_1) + \cdots + f_n^j(t_n)$.

Proof. The integral on the right side is

$$\int_{0 \leqq t_i \leqq 1} A(x^1, \cdots, x^n) \det \left| \frac{d f_i^j(t_i)}{d t_i} \right| dt_1 \cdots dt_n .$$

Let us substitute $f_i(t)$ by partially linear curves $g_i(t)$ whose corners are $g_i(k/N) = f_i(k/N)$, $k = 1, 2, \cdots, N$. For an arbitrarily given positive number δ , we can choose N sufficiently large such that $|f_i^j(t) - g_i^j(t)| < \varepsilon$ and $\left| \frac{d f_i^j(t)}{dt} - \frac{d g_i^j(t)}{dt} \right| < \varepsilon$; consequently we get

$$\left| A(x(f)) \det \left| \frac{d f_i^j(t_i)}{d t_i} \right| - A(x(g)) \det \left| \frac{d g_i^j(t_i)}{d t_i} \right| \right| < \delta$$

and error of the integral (2) and that substituted $g_i(t_i)$ thereinto is less than δ . Then, on account of (1) our result follows.

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VECTOR-GROUP IN REAL EUCLIDEAN SPACE

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We shall describe in this paper an elementary proof of the theorem which has also been proved in this volume by Prof. Iwamura, Messrs. M. Kuranishi and T. Hayashida.

We denote "free vectors" in an n-dimensional real euclidean space R_n by $x, y, z, \ldots, a, b, c, \ldots\ldots$, and the corresponding points in R_n by the same symbols, i.e., "a point x" means the point which is located by the free vector x starting from the original point o previously determined in R_n . The distance between any two points x and y is defined by the euclidean one, i.e., $|x - y|$. We shall prove in this paper the following Theorem and Corollary.

THEOREM. Let M be a real euclidean vector-group in R_n and contain a continuum K . Then M contains the whole straight-line through any two distinct points of K .

COROLLARY. Let M be a real euclidean vector-group in R_n and let any two points of M be connected by a continuum in M . Then M coincides with a real linear vector-group.

We shall prove the theorem by the induction with respect to the dimension-number n of R_n . If $n = 1$, the theorem is evident. Suppose $n > 1$.

LEMMA 1. Let K be any continuum in M . We define K' as the aggregate of all the points $x - y + z$, where x, y and z run throughout K . Then K' is also a continuum in M and $K \subset K'$.

The proof is immediate. We are going to prove that the straight-line segment joining any two distinct points a and b of K is contained in $K'' = (K')'$. As K is connected, a and b can be connected for any positive ε by an ε-chain with its points of joint all belonging to K . This chain can be represented by

$$x(t) ; \quad 0 \leqslant t \leqslant 1 ,$$

where $x(t)$ is a continuous curve in $0 \leqslant t \leqslant 1$, with its points of joint $x(t_i)$: $0 = t_0 < t_1 < t_2 < \ldots < t_m = 1$ all belonging to K and the parts $x(t)$, $t_i \leqslant t \leqslant t_{i+1}$, $i = 0, 1, 2, \ldots, m-1$ are all straight-line segments. Moreover $|x(t_{i+1}) - x(t_i)| < \varepsilon$, for $i = 0, 1, 2, \ldots, m-1$.