by the same argument as above, we have

$$\overline{\lim_{n\to\infty}}\ \varphi_{x_n+y_n}(\alpha) \le \varphi_y(\alpha).$$

Combining this with above result, we get

$$\lim_{n\to\infty} \varphi_{x_n+y_n}(\alpha) = \varphi_y(\alpha).$$

This completes the proof.

Theorem 5. If $x(t)$ has the unit asymptotic distribution function and $y(t)$ has an asymptotic distribution function φ_y, then $x(t)+y(t)$ has also an asymptotic distribution function φ_y.

Proof. By the same way as in the proof of Theorem 4, we have

$$\overline{\lim_{T\to\infty}}\ \frac{1}{2T} m E_t[-T\le t\le T, x(t)+y(t)>\alpha]$$
$$\le 1-\varphi_y(\alpha-\varepsilon)+1-\varphi_x(\varepsilon) = 1-\varphi_y(\alpha-\varepsilon).$$

Since $\varepsilon>0$ is arbitrary, if α is a continuity point of φ_y, then we have

$$\overline{\lim_{T\to\infty}}\ \frac{1}{2T} m E_t[-T\le t\le T, x(t)+y(t)>\alpha]$$
$$\le 1-\varphi_y(\alpha),$$

that is

$$1-\underline{\lim_{T\to\infty}}\ \frac{1}{2T} m E_t[-T\le t\le T, x(t)+y(t)<\alpha]$$
$$\le 1-\varphi_y(\alpha),$$

or

$$\underline{\lim_{T\to\infty}}\ \frac{1}{2T} m E_t[-T\le t\le T, x(t)+y(t)<\alpha] \ge \varphi_y(\alpha).$$

Analogously we have

$$\overline{\lim_{T\to\infty}}\ \frac{1}{2T} m E_t[-T\le t\le T, x(t)+y(t)<\alpha] \le \varphi_y(\alpha)$$

That is

$$\lim_{T\to\infty} \frac{1}{2T} m E_t[-T\le t\le T, x(t)+y(t)<\alpha] = \varphi_y(\alpha).$$

This completes the proof.

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A NOTE ON GENERATORS OF COMPACT LIE GROUPS

By Hiraku TOYAMA and Masatake KURANISHI.

H. Auerbach has obtained the following theorem [1] :

THEOREM: Let G be a (connected) compact Lie group, and for any integer k let

$$M(x,y,k)=\{p;\ p=\prod^{k} v_i,\quad v_i=x^{n_i} \text{ when } i \text{ is odd},\quad v_i=y^{n_i} \text{ when } i \text{ is even}\}$$

$$M(x,y)=\bigcup_{k=1}^{\infty} M(x,y,k)$$

Then there exist x and y such that $G=\overline{M(x,y)}$.

Here arises a question: Is there any integer k such that $G=\overline{M(x,y,k)}$. The affirmative answer for this question can easily be obtained. Let $f(G)$ be the minimum of such k. The next problem, to determine $f(G)$ for each compact Lie group, is not yet solved for the writers, but it can be seen

$$f(G) \geqq \dim(G)/\mathrm{rank}(G)$$

where rank(G)* is the dimension of a maximal abelian subgroup of G .

This note will contain the proofs of these two propositions.

For any element x of G , let $T(x)$ be the abelian closed subgroup of G generated by x , and put

(1) $$H(x,y,k)=\{p;\ p=\prod_{i=1}^{k} w_i,\quad w_i\in T(x) \text{ when } i \text{ is odd and } w_i\in T(y) \text{ when } i \text{ is even}\}$$

(2) $$H(x,y) = \bigcup_{k=1}^{\infty} H(x,y,k)$$

Then it is clear that

(3) $$H(x,y,k) \subseteq \overline{M(x,y,k)}$$

If $G=\overline{M(x,y)}$ and if $T(x)$ and $T(y)$ are connected, we shall say that x and y constitute a pair of generators of G . The existence of such x and y is proved in [1].

(1) When G is simply connected: Take a pair of generators x, y of G . Then $H(x,y)$ is an arc-wise connected subgroup of G and everywhere dense in G . It follows from these that $H(x,y) = G$ (for the proof see [2]). From