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DECOMPOSITION OF A k-COVECTOR WITH RESPECT TO A VECTOR AND COMPUTING ITS COMASS

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I. Introduction

A smooth differential form Φ on a Riemannian manifold which is closed and has comass one is called a calibration. Corresponding to a calibration is a geometry of minimal surfaces (cf. [H], [HL]).

The constant coefficient calibrations have been studied deeply by R. Harvey, B. Lawson, F. Morgan, J. Dadok ... Many beautiful constant coefficient calibrations and corresponding geometries were constructed in the nice paper of R. Harvey and B. Lawson [HL], for example Special Lagrangian, Associative, Coassociative, Cayley calibrations ... Computing the comass of a k-differential form is quite difficult, even in the simplest cases, the cases of k-covectors viewed as paralell differential forms. The known calibrations are not much, especially the calibrations of high degree. The such well-known calibrations are only Complex Line, Special Lagrangian, power of Kähler forms (see [DHM], [HL]).

The Associative and Coassociative calibrations (see [HL]) on $\boldsymbol{R}^7$ have many beautiful properties, and between them there is a relationship

$$*\varphi = \psi,$$

where φ is Associative calibration, and ψ is Coassociative calibration on $\boldsymbol{R}^7$.

Moreover,

$$\varphi(\eta)^2 + \bar{\psi}(\eta)^2 = 1 \quad \text{for all } \eta \in G(3, \boldsymbol{R}^7),$$

and hence

$$G(\varphi) = G_0(\bar{\psi}).$$

This paper gives a method to compute the comass of some classes of k-covectors, describes the set of all 3-covectors have comass one on $\boldsymbol{R}^8$, whose faces contain a $SLAG$ face (this set is denoted by $F^*(SLAG)$), and constructs new calibrations on $\boldsymbol{R}^{4n-1}$: General Associative and General Coassociative calibrations. The method bases on the decomposition of a covector Φ with respect to a vector $e \in \operatorname{span} \Phi^*$,

$$\Phi = e^* \wedge \varphi + \psi,$$

where $\varphi \in \bigwedge^{k-1}(e^{\perp})$; $\psi \in \bigwedge^{k}(e^{\perp})$.

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