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ON THE VALUE DISTRIBUTION OF $ff^{(k)}$

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Abstract

Let f be a transcendental entire function. In this paper we will prove that if f is of finite order, then there exists at most one integer $k \geqq 2$ such that $ff^{(k)}$ may have non-zero and finite Picard exceptional value. We also give a class of entire functions which have no non-zero finite Picard values. If f is a transcendental meromorphic function, we obtain that for non-negative integers $n, n_1, \cdots, n_k$ with $n \geqq 1$, $n_1 + \cdots + n_k \geqq 1$, if $\delta(o, f) > 3/(3n + 3n_1 + \cdots + 3n_k + 1)$, then $f^n (f')^{n_1} \cdots (f^k)^{n_k}$ has no finite non-zero Picard values.

I. Introduction and main results

Let f be a transcendental meromorphic function. In 1959, W. K. Hayman [4] proved that if n is an integer satisfying $n \geqq 3$, then $f^n f'$ takes every non-zero complex value a infinitely often. He conjectured [5] that this remains valid for $n=1$ and $n=2$. The case $n=2$ was settled by E. Mues [9] on 1979. The case $n=1$ is still open.

J. Clunie [3] proved that Hayman's conjecture is true when f is entire and $n=1$. W. Hennekemper [7] extended Clunie's result and proved

$$(1) \qquad T(r, f) \leqq \left(4 + \frac{1}{k+1}\right)\left\{\bar{N}(r, f) + \bar{N}\left(\frac{1}{(f^{k+1})^{(k)} - c}\right)\right\} + S(r, f)$$

for $k \in N$, $c \in C - \{0\}$, where the argument used here is based on the Nevanlinna theory, its associated standard symbols and notations, see, e.g. [6]. Particularly, $S(r, f)$ will be used to denote any quantity that satisfies $S(r, f) = o\{T(r, f)\}$ as $r \to \infty$ and $r \notin E$ with E being a set of r values of finite linear measure. W. Bergweiler and A. Eremenko [2] proved this for functions of finite order. Recently, Q. Zhang [16] extended Hennekemper's result (1) for $k=1$ and c is replaced by any small function $a(z)$ $(\not\equiv 0)$ of f, i.e. $a(z)$ satisfies $T(r, a) = S(r, f)$. W. Bergweiler [1] proved that if f is a transcendental meromorphic function of finite order and if a is a polynomial which does not vanish identically, then $ff' - a$ has infinitely many zeros.

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