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KODAI MATH. J.
17 (1994), 390—394

A NOTE ON GORENSTEIN DIMENSION AND THE AUSLANDER-BUCHSBAUM FORMULA

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Introduction

In [1] a generalization of the classical Auslander-Buchsbaum formula is made. Namely, if R is a noetherian local commutative ring and M is an R-module of finite Gorenstein dimension, then one has

$$G-\dim M+\operatorname{depth} M=\operatorname{depth} R$$

where G-dim M denotes the Gorenstein dimension of the module M. It is worth noting an important fact in this setting: If M has finite Gorenstein dimension then

$$G-\dim M=\sup\{t;\operatorname{Ext}_R^t(M,R)\neq 0\}$$

This equality leads us to define the so callled *weak Gorenstein dimension.* Let R be a commutative noetherian ring. Hereafter the notation $\operatorname{Ext}_R^t(M,R)$ will be abbreviated to $\operatorname{Ext}^t(M,R)$, unless otherwise specified.

DEFINITION. We say that a finitely generated R-module M has weak Gorenstein 0 if $\operatorname{Ext}^t(M,R)=0$ for all $t>0$. If $k>0$, we say that M has weal Gorenstein dimension (written w.g.d$(M)=k$) if $\operatorname{Ext}^t(M,R)=0$ for all $t>k$ while $\operatorname{Ext}^k(M,R)\neq 0$. Also we put w.g.d$(M)=\infty$ if $\operatorname{Ext}^t(M,R)\neq k$ for all $k=0,1,2,\ldots$.

From this definition we see that every projective module P has w.g.d.$(P)=0$, and it turns out that the class of all zero-dimensional modules plays an important role in our stũdy so we also give

DEFINITION. We denote by $\mathcal{C}_0(=\mathcal{C}_0(R))$ the family of finitely generated R-modules M for which w.g.d$(M)=0$.

Let R be a local ring, i.e. there is a unique maximal ideal $\mathcal{M}$. The depth of a finitely generated R-module M can be defined by the formula

$$\operatorname{depth}(M)=\inf\{k:\operatorname{Ext}^k(R/\mathfrak{M},M)\neq 0\}.$$

Now we announce the main results in this note.

THEOREM A. *Let R be a local noetherian ring. Assume that every module in the class is reflexive. Then*

$$\text{w.g.d}(M)+\operatorname{depth}(M)=\operatorname{depth}(R)$$

Received May 30, 1993.