Q. DING
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ON SPECTRAL CHARACTERIZATIONS OF MINIMAL HYPERSURFACES IN A SPHERE

BY QING DING

Abstract

Let M be a closed minimal hypersurface in an Euclidean sphere $S^{n+1}(1)$. We first prove that a minimal isoparametric hypersurface M in a 4-dimensional sphere is completely determined by its spectrum $\mathrm{Spec}^p(M)$, here $p\in\{0,1,2,3\}$. In higher dimensional sphere, we prove that if $\mathrm{Spec}^p(M)=\mathrm{Spec}^p(M_{m,n-m})$ for $p=0,1$, where

$$M_{m,n-m}=S^m\left(\sqrt{\frac{m}{n}}\right)\times S^{n-m}\left(\sqrt{\frac{n-m}{n}}\right)$$

is a Clifford torus, then M is $M_{m,n-m}$. Furthermore, we prove that $M_{n,n}\to S^{2n+1}(1)(n\geqq 4)$ is also characterized by $\mathrm{Spec}^p(M_{n,n})$ for some $p=p(n)$.

§1. Introduction

For a smooth compact, oriented Riemannian manifold M of dimension n, let $\Lambda^p(M)$ denote the space of C^∞ differential forms of degree $p=0, 1, \cdots, n$ with real coefficients. The Laplace operator Δ of M acting on functions has a natural generalization to $\Lambda^p(M)$. In the theory of spectrum of Laplace operator on $\Lambda^p(M)$, one can see that the interplay among analysis, topology and geometry is even striking (e.g., see [6]). We denote by $\mathrm{Spec}^p(M)$ the spectrum of Laplace operator on $\Lambda^p(M)$.

It is interesting to see the relation of $\mathrm{Spec}^p(M)$ and the geometry on M, which gives rise to the following old question: Does $\mathrm{Spec}^p(M)$ determine the geometry of Riemannian manifold M? The answer to this problem in general case is negative. This is a consequence of the counter example which is given by Milnor in [10]. So the problem is divided into two directions. One direction is to find new counter examples. A series studies along this line have been done by Vigneras [13], Ikeda [8] and others. Another direction is to give an affirmative answer for a special Riemannian manifold. The studies of this direction have also been done by Berger [1], Patodi [11], Tanno [12] and many others.

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