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# PERIOD-PRESERVING VARIATION OF A RIEMANN SURFACE

Dedicated to Professor Mitsuru Nakai on his sixtieth birthday

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## Introduction.

In the Teichmüller space of a given surface, the sub-locus, consisting of all marked Riemann surfaces which admit holomorphic abelian differentials having the prescribed periods, plays several important roles in the theory of Riemann surfaces.

In this note, we will introduce explicit local parameters at a generic point of this sub-locus. They indicate how zeros of differentials with prescribed periods vary on the surfaces.

## §1. The fundamental surgery.

Let $\alpha=re^{i\theta}\in\boldsymbol{C}$ ($r>0$, $0\leqq\theta\leqq 2\pi$) be arbitrarily given. Then a surgery of $\boldsymbol{C}$ which preserves the differential $zdz$ can be defined as follows: First, set $D(\alpha)=\boldsymbol{C}-\{z=t\cdot r^{1/2}e^{i\theta/2}\mid -1\leqq t\leqq 1\}$. Then $D(\alpha)$ is mapped conformally onto $D(-\alpha)=\boldsymbol{C}-\{w=it\cdot r^{1/2}e^{i\theta/2}\mid -1\leqq t\leqq 1\}$ by the mapping

$$w=\frac{1}{2}\left(\zeta-\frac{\alpha}{\zeta}\right) \quad \text{with} \quad z=\frac{1}{2}\left(\zeta+\frac{\alpha}{\zeta}\right),$$

namely,

$$w=F(z)=z\cdot\left(1-\frac{\alpha}{z^2}\right)^{1/2},$$

where we take the branch of $(1-\alpha/z^2)^{1/2}$ such that $1^{1/2}=1$.

Then $w=F(z)$ maps $D(\alpha)$ onto $D(-\alpha)$. Furthermore, a simple computation shows that

$$wdw=zdz.$$

DEFINITION. We call this surgery the fundamental surgery (of the differential $zdz$) at $z=0$ with respect to $\alpha$.

*Remark.* This is a kind of Schiffer's interior variation, and can be regarded also as the branch-point variation with respect to the branched covering projec-

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