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SELF-MAPS OF SPHERE BUNDLES II

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§ 1. Introduction.

Let E be an oriented orthogonal q-sphere bundle over a connected finite CW-complex B. A fibre-preserving map $f: E \to E$ is said to have degree m when its restriction to some fibre is a map of degree m in the familiar sense; because B is path-connected it makes no difference which fibre we choose. Given E and an integer m is there a fibre-preserving map $f: E \to E$ of degree m? This question was put to me in 1971 by I. M. James, and in [2] there are some answers in fairly general situations. In the present paper I consider in more detail the special case where B is a sphere S^{r+1}. We first make some simple observations.

The identity map has degree 1, and when q is even E always admits a fibre-preserving map of degree -1; this is because the antipodal map $a: S^q \to S^q$ commutes with the action of the group $SO(q+1)$ of rotations in $\boldsymbol{R}^{q+1}$ and therefore extends to a fibre-preserving map: it would be interesting to know what happens when E is a general oriented q-spherical fibration with q even. If E admits fibre-preserving maps of degrees m, n then their composite is a fibre-preserving map of degree mn. Apart from this, nothing is very obvious.

Let $\pi: E \to B$ be the projection. Then when E has a cross-section s the composite $s\pi: E \to E$ is a fibre-preserving map of degree 0. In [2] the converse is proved, namely that if E admits a fibre-preserving map of degree 0 then E has a cross-section. (It is not the case that every fibre-preserving map $f: E \to E$ of degree 0 is homotopic through fibre-preserving maps to one of the form $s\pi$ for some cross-section s, but if B is covered by k contractible open subsets then f^k is homotopic through fibre-preserving maps to $s\pi$ for some cross-section s.) Some of the main results of [2] describe the structure of the set $A(E)$ of integers m such that E admits a fibre-preserving map of degree m. In the present paper we prove some results that allow us to estimate $A(E)$ when $B = S^{r+1}$.

If E^* is a fibre bundle over S^{r+1} with fibre F^* let $o(E^*)$ be the obstruction to a cross-section of E^*, as defined in § 2 below. From now on let $B = S^{r+1}$. In § 2 we show that a necessary condition for there to be a fibre-preserving map $E \to E$ of degree m is that $\phi_m o(E) = o(E)$. Here $\phi_m: \pi_r S^q \to \pi_r S^q$ is induced by a map of degree m on S^q.

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