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THE INVARIANT PSEUDO-METRIC RELATED TO NEGATIVE PLURISUBHARMONIC FUNCTIONS

Dedicated to Professor Tadashi Kuroda on his 60th birthday

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Introduction.

In [1], using a family of negative plurisubharmonic functions on a complex manifold M, the author defined an invariant pseudo-metric P^M on M whose indicatrices are always pseudoconvex domains in the holomorphic tangent spaces. On the other hand, Klimek [5] defined an extremal plurisubharmonic function g_p^M with pole at a given point p of M.

The aim of the present note is to clarify the relationship between P^M and g_p^M (Proposition 2.4), and to simplify the original construction of P^M in [1] (Lemma 2.1, Corollary 2.5). We also show that P^M is a higher-dimensional generalization of the pseudo-metric $c_\beta^z|dz|$ induced from the capacity $c_\beta^z=\exp(-k_\beta^z)$ on an open Riemann surface M (cf. [11]), where $k_\beta^z(p)$ is the Robin constant at a point p of M with respect to a local coordinate z around p (Proposition 3.1). Finally, we derive some results related to the pseudo-metric P^M for Riemann surfaces M.

§1. Klimek's extremal plulisubharmonic functions.

Let p be a point of a complex manifold M. We denote by $PS^M(p)$ the family of all $[-\infty, 0)$-valued plurisubharmonic functions f on M such that the function $f-\log\|z\|$ is bounded from above in a deleted neighborhood of p for some holomorphic local coordinate z with $z(p)=0$. We note that every $f\in PS^M(p)$ takes the value $-\infty$ at p, and that $PS^M(p)$ always contains the constant function $-\infty$. The definition of the family $PS^M(p)$ does not depend on the choice of the coordinate z with $z(p)=0$. According to Klimek [5], we define the extremal function g_p^M on M by

$$g_p^M(q)=\sup\{f(q)\,;\ f\in PS^M(p)\}$$

for $q\in M$.

We quote from [5] some results on g_p^M. In [5], Klimek dealt with the case when M is a domain in $\boldsymbol{C}^m$. However, one can see that these assertions hold also for prescribed manifolds M.

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