G. PITIS
KODAI MATH. J.
9 (1986), 327—333

ON SOME SUBMANIFOLDS OF A LOCALLY PRODUCT MANIFOLD

BY GHEORGHE PITIS

An investigation of properties of submanifolds of the almost product or locally product Riemannian manifolds has been started in the last years, many interesting results being obtained. So, Okumura [8], Adati and Miyazawa [1], Miyazawa [7], studied the hypersurfaces of such manifolds, Adati [2], defined and studied the invariant, anti-invariant and non-invariant submanifolds, while Bejancu [4], analyzed the semi-invariant submanifolds which are corresponding to CR-submanifolds of a Kaehler manifold [3].

The purpose of this paper is to give some properties of the anti-invariant and semi-invariant submanifolds, by using cohomology groups.

In § **1** we recall the definition of these submanifolds and some known results, already. An example of semi-invariant submanifold is given.

In § **2** we associate to a semi-invariant submanifold a de Rham cohomology class (as in [5] for CR-submanifolds) and we obtain a connection between the properties of the invariant and anti-invariant distributions and the cohomology of the submanifold (theorem 2.2).

The stability of some anti-invariant submanifolds of a locally product Riemannian manifold is studied in § **3** and we give algebraic conditions for stability.

§ 1. Anti-invariant and semi-invariant submanifolds of a locally product Riemannian manifold.

Let $(\tilde{M}, g, F)$ be a C^∞-differentiable almost product Riemannian manifold, where g is a Riemannian metric and F is a non-trivial tensor field of type (1.1). Moreover g and F satisfy the following conditions

$$(1.1) \qquad F^2=I\,, \quad (F\neq \pm I)\,; \quad g(FX, FY)=g(X, Y)\,, \quad X, Y\in \mathfrak{X}(\tilde{M})$$

where I is the identity and $\mathfrak{X}(\tilde{M})$ is the Lie algebra of vector fields on $\tilde{M}$.

We denote by $\tilde{\nabla}$ the Levi-Civita connection on $\tilde{M}$ with respect to g and furthermore we assume that $\tilde{M}$ is locally product, that is

$$(1.2) \qquad \tilde{\nabla}_X F=0 \qquad X\in \mathfrak{X}(\tilde{M})\,.$$

Let M be a Riemannian manifold isometrically immersed in $\tilde{M}$ and denote

Received June 26, 1985