T. KATO
KODAI MATH. J.
7 (1984), 120—132

ON THE ORDER OF AUTOMORPHISM GROUP OF A COMPACT BORDERED RIEMANN SURFACE OF GENUS FOUR

Dedicated to Professor Mitsuru Ozawa on his 60th birthday

BY TAKAO KATO

§0. Introduction. For non-negative integers g and k $(2g+k-1\geqq 2)$, let $N(g, k)$ be the maximum of the orders of the automorphism groups of compact bordered Riemann surfaces of genus g having k boundary components. Oikawa [9] proved that every automorphism group of a compact bordered Riemann surface is isomorphic to a subgroup of the automorphism group of a compact Riemann surface of the same genus and that $N(g, k)$ is equal to the maximum of the order of the automorphisms groups of k-times punctured compact Riemann surfaces of genus g. Hurwitz [3] proved that $N(g, 0)\leqq 84(g-1)$. For infinitely many values of g, $N(g, 0)$ were determined by [1, 6, 7, 8]. But, for infinitely many g, $N(g, 0)$ are not known. For every $g\geqq 0$, $N(g, 1)$, $N(g, 2)$ and $N(g, 3)$ were determined by the author [4], for every $k\geqq 0$, $N(0, k)$, $N(1, k)$, $N(2, k)$ and $N(3, k)$ were determined by [2, 9, 11, 12] and for many other special pairs of g and k, $N(g, k)$ were determined by Ouchi [10]. In this paper we shall determine $N(4, k)$ for every $k\geqq 0$. Wiman [14] showed the equations of all the compact Riemann surfaces of genus 4 which have non-trivial automorphism groups and proved that $N(4, 0)=120$. To determine $N(4, k)$, we shall study subgroups of groups which Wiman showed.

The author wishes to represent his thanks to Professor Accola who showed Wiman's paper [13, 14] and gave some advice to him and to Mr. Nakagawa who read the manuscript of this paper and pointed out many errors.

§1. Lemmas: Let S be a compact Riemann surface of genus 4 and let G be a group of automorphisms of S. S/G has the conformal structure induced by the conformal structure of S such that the natural projection π of S onto S/G is holomorphic. Then, there are at most finite number of points $P_1, \cdots, P_t$ on S/G over which π is ramified with multiplicities $\nu_1, \cdots, \nu_t$ $(\nu_j\geqq 2)$, respectively. Then Riemann-Hurwitz's relation shows

$$6/N=2\tilde{g}-2+\sum_{j=1}^{t}(1-1/\nu_j),$$

Received April 12, 1983