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A NOTE ON THE PRODUCT OF MEROMORPHIC FUNCTIONS AND ITS DERIVATIVES

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Abstract

It is shown that if f is an even or odd transcendental meromorphic function and if c is any even meromorphic function which does not vanish identically and satisfies $T(r,c) = o(T(r,f))$ as $r \to +\infty$, then $ff' - c$ has infinitely many zeros.

1. Introduction and our main results

In 1959, W. K. Hayman [5] proved that

Theorem A. *If n is an integer greater than or equal to 3 and f is a transcendental meromorphic function, then $f^n f'$ takes every non-zero complex number infinitely many times.*

Later, he conjectured [6] that this remains valid for the cases $n = 1, 2$. In 1979, E. Mues [7] proved the case $n = 2$ and the conjecture was proven by A. Eremenko and W. Bergweiler [2] in 1995 and independently by H. H. Chen and M. L. Fang [3].

In 1994, Yik-Man Chiang asked W. Bergweiler whether $ff' - c$ has infinitely many zeros if f is a transcendental meromorphic function and if c is a meromorphic function which does not vanish identically and satisfies $T(r,c) = o(T(r,f))$ as $r \to +\infty$. In [8], Q. D. Zhang studied the value distribution of $\varphi(z)f(z)f'(z)$ and obtained the following theorem.

Theorem B. *If f is a transcendental meromorphic function and φ is a non-zero meromorphic function such that $T(r,\varphi) = S(r,f)$ as $r \to +\infty$, then*

$$T(r,f) < \frac{9}{2}\overline{N}(r,f) + \frac{9}{2}\overline{N}\left(r, \frac{1}{\varphi f f' - 1}\right) + S(r,f).$$

By this, we have

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