B.-Y. CHEN
KODAI MATH. J.
10 (1987), 93—101

SOME ESTIMATES OF TOTAL TENSION AND THEIR APPLICATIONS

BY BANG-YEN CHEN

Abstract

In this note, we give two best possible estimates of the total tension for a smooth map. Such estimates are established in terms of order of the map. Applications of such estimates to isometric immersions and to spectral geometry are given by applying an inequality obtained in [3].

1. Introduction.

Let M be a compact submanifold of a Euclidean m-space E^m. By applying the induced metric on M, the author introduced in [2] the notion of order of the submanifold. The notion of order is known to be closely related with the differential geometry of the submanifold (cf. [4]). In [5, 6] such notion was generalized to smooth maps of a compact Riemannian manifold into E^m. Some relations between the total tension and the order were obtained in [5, 6].

In this note, we will obtain two more relations between the total tension and the order of a map. Such relations are applied to obtain a best possible estimate of the total mean curvature of a spherical submanifold. By using a best possible inequality derived in [3], such relations were then applied to obtain some best possible eigenvalue estimates for minimal submanifolds in rank-one symmetric spaces.

2. Order of a Map.

Let M be a compact n-dimensional Riemannian manifold and Δ the Laplacian of M acting on the space $C^\infty(M)$ of smooth functions. Then Δ has an infinite discrete sequence of eigenvalues:

$$0=\lambda_0<\lambda_1<\lambda_2<\cdots<\lambda_k<\cdots\uparrow\infty.$$

For each k ($k=0, 1, 2, \cdots$), the eigenspace $V_k=\{f\in C^\infty(M): \Delta f=\lambda_k f\}$ is finite-dimensional. With respect to the inner product $(f, g)=\int_M fg\, dV$ on $C^\infty(M)$, the

Received July 11, 1986