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KODAI MATH. J.
10 (1987), 49—54

REPRESENTATION OF ADDITIVE FUNCTIONALS ON MUSIELAK-ORLICZ SPACE OF VECTOR-VALUED FUNCTIONS

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In the paper [4] Hiai proved theorems on representation of additive functionals on vector-valued normed Köthe spaces. His theory, as is shown by the Example 2., does not contain an important and large class of non-solid Orlicz and Musielak-Orlicz spaces of vector-valued functions. Nevertheless, a very interesting idea of the proofs is so universal that it can be applied in the above case as well. It is necessary to change only proofs of Lemma 3.1 and Theorem 3.4 in which the following assumption

$$\|f(t)\|_X \leqq \|g(t)\|_X \quad \text{for almost all } t \text{ implies } \|f\|_M \leqq \|g\|_M$$

is essential. Therefore, in order to avoid the repetition of argumentations presented in the paper [4], this note contains the modifications of Lemma 3.1 and Theorem 3.4. Then representation theorems for additive lower semicontinuous and continuous functionals are presented as a conclusion. Moreover, it is worth to notice that the representation theorem for bounded linear functionals, considered also by Hiai, in these spaces was elementarily proved in a particular case by Kozek (see [8]).

1. Introduction. Let (T, Σ, μ) be a positive, σ-finite and complete measure space. $(X, \|\cdot\|_X)$ denotes a separable real Banach space.

DEFINITION 1. A function $M: X\times T\rightarrow[0, \infty]$ is said to be an $\mathcal{N}$-function, iff

a) M is $\mathcal{B}\times\Sigma$-measurable, where $\mathcal{B}$ denotes the σ-algebra of Borel subsets of X,

b) $M(\cdot, t)$ is even, convex, lower semicontinuous, continuous at zero and $M(0, t)=0$ for $a.a.$ $t\in T$,

c) $\lim_{\|x\|_X\rightarrow\infty} M(x, t)=\infty$ $a.e.$ in T.

Let us assume that $\mathcal{N}$-function M satisfies the so-called Condition B, which can be also formulated in the following simple form (see [15] Remark 1.5)

Received April 8, 1986