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Boundedness of global solutions of one dimensional quasilinear degenerate parabolic equations

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1. Introduction.

Let $\Omega = (-L, L)$ be a bounded open interval in $\boldsymbol{R}$. In this paper we shall consider the one-dimensional Dirichlet problem

$$\partial_t \beta(u) = u_{xx} + f(u) \quad \text{in} \quad (x,t) \in \Omega \times (0,T) \tag{1.1}$$

$$u(\pm L, t) = 0 \quad \text{on } t \in (0,T) \tag{1.2}$$

$$u(x,0) = u_0(x) \quad \text{on } x \in \Omega \tag{1.3}$$

where $\partial_t = \partial/\partial t$ and $\beta(v)$, $f(v)$ with $v \geq 0$ and $u_0(x)$ are nonnegative functions.

Equation (1.1) describes the combustion process in a stationary medium in which the thermal conductivity $\beta'(u)^{-1}$ and the volume heat source $f(u)$ are depending in a nonlinear way on the temperature $\beta(u) = \beta(u(x,t))$ of the medium.

Throughout this paper we assume

(A1) $\beta(v), f(v) \in C^\infty(\boldsymbol{R}_+) \cap C(\bar{\boldsymbol{R}}_+)$ where $\boldsymbol{R}_+ = (0,\infty)$ and $\bar{\boldsymbol{R}}_+ = [0,\infty)$, $\beta(v) > 0$, $\beta'(v) > 0$, $\beta''(v) \leq 0$ and $f(v) > 0$ for $v > 0$, $\lim_{v\to\infty} \beta(v) = \infty$, $f \circ \beta^{-1}(v)$ is locally Lipschitz continuous in $v \geq 0$.

(A2) $u_0(x) \geq 0$, $\in C(\bar{\Omega})$ and $u_0(\pm L) = 0$ (compatibility condition).

With these conditions above Dirichlet problem has a unique local solution $u(x,t) \geq 0$ (in time) which satisfies (1.1)$\sim$(1.3) in a weak sense (e.f.- Aronson-Crandall-Peletier [**3**], Bertsch-Kersner-Peletier [**4**], Ladyzenskaja, et al. [**11**], Oleinik et al. [**13**]). The definition of "weak" solutions is given in Section 2.

Let $s(x)$ be the principal eigensolution of $-\partial^2/\partial x^2$ in $(-L,L)$ with Dirichlet boundary conditions (s is normalized: $s > 0$ in $\Omega, \int_\Omega s(x)\,dx = 1$) and λ be the first eigenvalue of this problem. We further assume the almost necessary condition to raise the blow-up (see Imai-Mochizuki [**9**]).

(A3) There exist a continuous function $g(\xi)$ of ξ and a $\xi_1 \geq 0$ such that

$$g(\xi) \leq f(\xi) - \lambda\xi \quad \text{in } \xi \geq 0, \tag{1.4}$$

$$\Gamma \equiv g \circ \beta^{-1} \text{ is convex in } (\beta(0), \infty), \tag{1.5}$$

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