J. Math. Soc. Japan
Vol. 48, No. 3, 1996

Sector theory and automorphisms for factor-subfactor pairs

Dedicated to Professor Masamichi Takesaki on his sixtieth birthday

By Hideki KOSAKI

(Received May 20, 1994)
(Revised Aug. 29, 1994)

1. Introduction.

The index theory ([**19**]) for II_1-factors was initiated by Jones about ten years ago. Since then tremendous progress has been made in the subject matter. Especially, classification of subfactors with small indices in the AFD (II_1) factors is of particular interest (see [**41, 42, 45, 46**] and also [**18, 21**]), and this makes it possible to study automorphisms for factor-subfactor pairs in details (see for example [**22, 33, 47**]).

On the other hand, the notion of an index has been generalized to wider classes of operator algebras (for example [**25, 37, 52**]). In Longo's approach on index theory ([**37, 38**]) for factors of type III, the notion of a sector plays a fundamental role. This notion originally occurred in Quantum Field Theory, and it has been proved extremely useful by recent works of Izumi and Longo ([**14, 15, 16, 39, 40**]).

In our previous papers [**1, 28**], we saw that sectors are also useful to analyze automorphisms for factor-subfactor pairs. Let $M \supseteq N$ be a factor-subfactor pair (with finite index), and $\theta \in Aut(M, N)$ be an automorphism for the pair. Let $\{M_k\}_{k=0,1,2,\cdots}$ be the Jones tower, and we assume that θ is already extended to the tower in the canonical way. Then, θ is called strongly outer ([**1**]) if, for $x \in M_k$, the commutation relation $yx = x\theta(y)$ for all $y \in N$ forces $x=0$, and in ([**28**]) we saw that the strong outerness is characterized by making use of relevant sectors. Namely, θ is strongly outer if and only if it does not appear (as an irreducible component) in $\bigsqcup_k (\rho\bar{\rho})^k$, where ρ is a sector (or an endomorphism) satisfying $N = \rho(M)$ (see § 3 for details). In terms of bimodules naturally attached to the inclusion $M \supseteq N$ in the Ocneanu approach ([**41, 42**]), this condition means that the $M-M$ bimodule canonically determined

This research was partially supported by Grant-in-Aid for Scientific Research (No. 06221257), Ministry of Education, Science and Culture.