

An example of self homotopy equivalences

Dedicated to Professor Teiichi Kobayashi on his 60th birthday

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1. Introduction.

Let us consider the principal fibre bundle

$$(1.1) \quad (P, q, B, G)$$

with structure group G . G acts on P freely.

Then one can consider the space of (unbased) G -equivariant self-homotopy equivalences of P , which we denote

$$(1.2) \quad \text{aut}_G(P).$$

We define

$$(1.3) \quad \mathcal{F}_G(P) = \pi_0(\text{aut}_G(P)).$$

We call this group the group of G -equivariant self-equivalences of the principal fibre bundle (1.1) (cf. [4, 5]).

Also one can consider the space of (unbased) self-homotopy-equivalences of P , which we denote

$$(1.4) \quad \text{aut}(P).$$

We define

$$(1.5) \quad \mathcal{F}(P) = \pi_0(\text{aut}(P)).$$

We call this group the group of self-equivalences of the space P .

We have a natural homomorphism from (1.3) to (1.5) forgetting the G -action

$$(1.6) \quad \mathcal{F}_G(P) = \pi_0(\text{aut}_G(P)) \longrightarrow \pi_0(\text{aut}(P)) = \mathcal{F}(P),$$

induced by the inclusion $\text{aut}_G(P) \rightarrow \text{aut}(P)$.

In [3, Problem 13, p. 206] the author has raised the following problem in 1988: when is the homomorphism (1.6) a monomorphism.

At this point no examples are known, where this homomorphism is not a monomorphism.