J. Math. Soc. Japan
Vol. 48, No. 2, 1996

Brelot spaces of Schrödinger equations

By Mitsuru NAKAI

(Received Sept. 1, 1993)
(Revised Jun. 10, 1994)

Consider a Radon measure μ of not necessarily constant sign on a subregion W of the Euclidean space $\boldsymbol{R}^d$ of dimension $d \geqq 2$. A function u on an open subset U of W is said to be *μ-harmonic* on U if u is continuous on U and satisfies the Schrödinger equation $(-\Delta+\mu)u=0$ on U in the sense of distributions. The family of μ-harmonic functions on open subsets of W determines a *sheaf* H_μ of functions on W (cf. § 1.1 below), i.e., $H_\mu(U)$ is the set of μ-harmonic functions on U. In order for us to be able to effectively discuss various global structures such as the Martin boundary related to the equation $(-\Delta+\mu)u=0$ on W, it is the least requirement for the sheaf H_μ to give rise to a Brelot harmonic space, or simply *Brelot space*, (W, H_μ) (cf. § 1.2). This paper concerns the question under what condition on μ the sheaf H_μ generates a Brelot space (W, H_μ). It was shown by Boukricha [3] for a positive measure μ and by Boukricha-Hansen-Hueber [4] for a signed measure μ that (W, H_μ) is a Brelot space if μ is of *Kato class* (cf. § 2.2). It is a natural question to ask whether for μ to be of Kato class is the widest possible condition for (W, H_μ) to be a Brelot space; specifically we ask whether μ is of Kato class if (W, H_μ) is a Brelot space. The answer to this question is given as follows:

MAIN THEOREM. *Although a Radon measure μ of constant sign being of Kato class is necessary and sufficient for the pair (W, H_μ) to be a Brelot space, a Radon measure μ of nonconstant sign being of Kato class is sufficient but not necessary in general for (W, H_μ) to be a Brelot space.*

We will give a self contained complete proof to the above assertion and actually more than described in the above statement as follows. We introduce a new notion of, what we call, a Radon measure of *quasi Kato class* (cf. § 3.2). We then have the following result:

THEOREM 1. *If μ is a Radon measure of quasi Kato class, then the pair (W, H_μ) is a Brelot space.*

To complete the present work the author was supported in part by Grant-in-Aid for Scientific Research, No. 06640227, Japanese Ministry of Education, Science and Culture.