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An exact sequence related to Adams-Novikov E_2-terms of a cofibering

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§1. Introduction.

Let E be a ring spectrum. The E-Adams spectral sequence with computable E_2-term is a useful tool for computing stable homotopy groups of spectra. We concern here about the BP-Adams spectral sequence $E_2=\mathrm{Ext}_{BP_*BP}^{s,*}(BP_*, BP_*(X))\Rightarrow\pi_{*-s}(X)$, in which BP is the Brown-Peterson spectrum at a prime p with the coefficient ring $BP_*=\pi_*(BP)=\boldsymbol{Z}_{(p)}[v_1, v_2, \cdots]$. For a spectrum X, we especially concern for the sphere spectrum S^0 and the Toda-Smith spectrum $V(n)$. In their paper [8], Miller, Ravenel, and Wilson have introduced the chromatic spectral sequence to compute the E_2-term of the above spectral sequence. Let $I_i=(p, v_1, \cdots, v_{i-1})$ be the invariant prime ideal of BP_*. They have defined BP_*BP-comodules A_i^j and LA_i^j by taking $A_i^0=\pi_*(BP)/I_i$, $LA_i^j=v_{i+j}^{-1}A_i^j$ and the short exact sequences $0\to A_i^j\to LA_i^j\to A_i^{j+1}\to 0$, inductively. The chromatic spectral sequence converging to $\mathrm{Ext}_{BP_*BP}^s(BP_*, BP_*/I_i)$ is given by the chromatic resolution

$$0 \longrightarrow BP_*/I_i \longrightarrow LA_i^0 \longrightarrow LA_i^1 \longrightarrow \cdots$$

associated to the short exact sequence above. Then they have defined "the universal Greek letter map" η: $\mathrm{Ext}_{BP_*BP}^s(BP_*, A_i^j)\to\mathrm{Ext}_{BP_*BP}^{s+j}(BP_*, BP_*/I_i)$ by taking the composition of the coboundary homomorphisms, and the Greek letter elements to be the image of η.

In this paper we also consider the Johnson-Wilson spectrum $E(n)$ for $n\geqq 0$ with $E(n)_*=\boldsymbol{Z}_{(p)}[v_1, v_2, \cdots, v_n, v_n^{-1}]$. Define $E(n)_*E(n)$-comodules B_i^j and LB_i^j in the same way as the case of BP by replacing the first step of the induction by $B_i^0=E(n)_*/I_i$ (see §3). Throughout the paper, we use the following notation for the Hopf algebroids appeared above:

$$(A, \Gamma)=(BP_*, BP_*BP)$$

and

$$(B, \Sigma)=(E(n)_*, E(n)_*E(n)=E(n)_*\otimes_A\Gamma\otimes_A E(n)_*).$$

Let L_nX (resp. C_nX) for a spectrum X denote the Bousfield localization