J. Math. Soc. Japan
Vol. 45, No. 4, 1993

G-s-cobordant manifolds are not necessarily G-homeomorphic for arbitrary compact Lie groups G

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(Received Sept. 4, 1992)

§1. Introduction.

The classical h-cobordism theorem and the s-cobordism theorem have played an important role in numerous aspects of geometric topology, including the classification of manifolds by surgery [35], [22], [3], [28], [21], [36], [34], [43], [5], [9], [23], [42].

In [1], we discussed equivariant versions of these theorems.

Let G be a compact Lie group and X a finite G-CW complex. S. Illman [14] defined the equivariant Whitehead group $\mathrm{Wh}_G(X)$ of X and the equivariant Whitehead torsion $\tau_G(f)$ for a G-homotopy equivalence $f: X \to Y$ between finite G-CW-complexes X, Y as an element of $\mathrm{Wh}_G(X)$. When $\tau_G(f)=0$, f is called a *simple G-homotopy equivalence*.

Let W be a compact smooth G-manifold whose boundary ∂W is the disjoint union $X \amalg Y$ of two closed G-invariant submanifolds. If the inclusion maps

$$i_X : X \longrightarrow W \quad \text{and} \quad i_Y : Y \longrightarrow W$$

are G-homotopy equivalences, then the triad $(W; X, Y)$ is called a *G-h-cobordism*.

When G is a finite group, W admits a unique smooth G-triangulation [15]. Accordingly the equivariant Whitehead torsion $\tau_G(i_X)$ is well-defined. On the other hand the investigation of T. Matumoto and M. Shiota [26] enables us to define the equivariant Whitehead torsion $\tau_G(i_X)$ even when G is a compact Lie group. Notice that $\tau_G(i_X)$ is often written as $\tau_G(W, X)$.

A G-h-cobordism $(W; X, Y)$ is called a *G-s-cobordism* when $\tau_G(i_X)$ vanishes. The two G-manifolds X and Y are then called *G-s-cobordant*.

We say that the G-s-cobordism theorem holds for a G-s-cobordism $(W; X, Y)$ if W is G-diffeomorphic to the product $X \times I$ rel X where I is the interval

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Partially supported by Grant-in-Aid for Scientific Research, The Ministry of Education, Science and Culture.