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Unicity theorems for the Gauss maps of complete minimal surfaces

Dedicated to Professor Shoshichi Kobayashi on his 60th birthday

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§ 1. Introduction.

Let M be a minimal surface in $\boldsymbol{R}^3$, or more precisely, a connected oriented minimal surface immersed in $\boldsymbol{R}^3$. By definition, the Gauss map G of M is the map which maps each point $p \in M$ to the unit normal vector $G(p) \in S^2$ of M at p. Instead of G, we study the map $g := \pi \cdot G : M \to \bar{\boldsymbol{C}} := \boldsymbol{C} \cup \{\infty\}$ for the stereographic projection π of S^2 onto $\bar{\boldsymbol{C}}$. The surface M is canonically considered as an open Riemann surface with a conformal metric and g is a meromorphic function on M. For a complete minimal surface in $\boldsymbol{R}^3$ g has many properties which have similarities to results in value distribution theory of meromorphic functions on $\boldsymbol{C}$. The author obtained some of them in the previous papers [**5**], [**6**] and [**7**]. The purpose of this paper is to give some unicity theorems for the Gauss map of minimal surfaces in $\boldsymbol{R}^3$ which are similar to the following theorem for meromorphic functions given by R. Nevanlinna ([**9**]):

THEOREM. *If two nonconstant meromorphic functions g and $\tilde{g}$ on $\boldsymbol{C}$ have the same inverse images for five distinct values, then $g \equiv \tilde{g}$.*

Let M and $\tilde{M}$ be two nonflat minimal surfaces in $\boldsymbol{R}^3$ and assume that there is a conformal diffeomorphism Φ of M onto $\tilde{M}$. Consider the maps $g := \pi \cdot G$ and $\tilde{g} := \pi \cdot \tilde{G} \cdot \Phi$, where G and $\tilde{G}$ are the Gauss maps of M and $\tilde{M}$ respectively. Suppose that there are q distinct points $\alpha_1, \alpha_2, \cdots, \alpha_q$ such that $g^{-1}(\alpha_j) = \tilde{g}^{-1}(\alpha_j)$ $(1 \leqq j \leqq q)$. The main result in this paper is stated as follows:

THEOREM I. *If $q \geqq 7$ and either M or $\tilde{M}$ is complete, then $g \equiv \tilde{g}$.*

For a particular case, we can show the following:

THEOREM II. *If $q \geqq 6$ and both of M and $\tilde{M}$ are complete and have finite total curvature, then $g \equiv \tilde{g}$.*

In Theorem I, the number seven is the best-possible. In fact, we can construct two mutually isometric complete minimal surfaces whose Gauss maps are dis-

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