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Vector valued invariants of prehomogeneous vector spaces

By Akihiko GYOJA

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0. Introduction.

0.1. Let G be a finite group acting linearly on a finite dimensional vector space V over a finite field $\boldsymbol{F}_q$. Let $\{v_0, \cdots, v_n\}$ be a complete set of representatives of V/G, $V_i = Gv_i$, $K_i = Z_G(v_i)$, $R: G \to GL(M)$ a complex representation, and M_i the set of K_i-fixed vectors in M. For each $m \in M_i$, there exists one and only one M-valued function $R_{i,m}$ on V_i such that $R_{i,m}(v_i) = m$ and $R_{i,m}(gv) = R(g)R_{i,m}(v)$ for $g \in G$ and $v \in V_i$. We extend $R_{i,m}$ by zero to the whole space V.

0.2. Our first problem is to know if the vector valued functions $R_{i,m}$ are similar in property to the complex powers of a relatively invariant polynomial function on a prehomogeneous vector space over the complex or real number field. (A rational representation of an algebraic group is called a prehomogeneous vector space, if the representation space has a Zariski open orbit.)

Let $V^\vee$ be the dual G-module of V, and define, in the same way as above, $\{v_0^\vee, \cdots, v_{n'}^\vee\}$, M_i', and M-valued functions $R'_{i',m'}$ $(1 \leqq i' \leqq n',\ m' \in M'_{i'})$ such that $R'_{i',m'}(gv^\vee) = R(g)R'_{i',m'}(v^\vee)$ for $g \in G$ and $v^\vee \in V^\vee$. As is easily seen, the Fourier transform of $R_{i,m}$ is a linear combination of these $R'_{i',m'}$'s. Provisionally in the introduction, let us assume that M_0 and M_0' are one dimensional and spanned by m_0 and m_0' respectively. Then the Fourier transform of R_{0,m_0} is a linear combination of $R'_{0,m_0'}$ and $\{R'_{i',m'} \mid 1 \leqq i' \leqq n',\ m' \in M'_{i'}\}$. Hence if m_0 and m_0' are given, the coefficient $c(R)$ of $R'_{0,m_0'}$ is uniquely determined.

Our first problem is, more precisely, the evaluation of the coefficient $c(R)$. See (2.4) and (3.4) for our result, where we calculate the value of $c(R)$ for some examples. In many cases, we can say from the value of $c(R)$ that the Fourier transform of R_{0,m_0} is, in fact, equal to $c(R)R'_{0,m_0'}$. See (2.6).

0.3. Our second problem is to understand character sum analogues of the Fourier transforms of complex powers of relative invariants of non-reductive prehomogeneous vector spaces in terms of the vector valued relative invariants