J. Math. Soc. Japan
Vol. 40, No. 1, 1988

On homotopy representations with the same dimension function

By Ikumitsu NAGASAKI

(Received June 9, 1986)

§ 0. Introduction.

This paper is concerned with classification of homotopy representations (up to G-homotopy). Let G be a finite group. A *homotopy representation* X of G is a finite dimensional G-CW-complex such that for each subgroup H of G, the H-fixed point set X^H is homotopy equivalent to a sphere S^m of dimension m which is equal to $\dim X^H$, or the empty set. T. tom Dieck and T. Petrie first introduced homotopy representations in [6] and studied their stable theory. Recently E. Laitinen studied the unstable theory of homotopy representations in [8] and showed that two homotopy representations X and Y are G-homotopy equivalent if and only if their dimension functions are equal and a certain invariant $D_n(X, Y)$ in the unstable Picard group $\mathrm{Pic}_n(G)$ vanishes, where $n=\mathrm{Dim}\, X=\mathrm{Dim}\, Y$.

T. tom Dieck studied the dimension functions of homotopy representations in [2]. In particular, he showed that the dimension function of a homotopy representation of a p-group G is equal to that of some linear G-sphere. (See also [7].) This result implies that the dimension functions of homotopy representations of a p-group are classified by the representation theory.

The purpose of this paper is to investigate the number $\mathrm{Num}(G, n)$ of G-homotopy types of homotopy representations with the same dimension function n.

In Section 1, we show that the number $\mathrm{Num}(G, n)$ is at most the order of $\mathrm{Pic}_n(G)$ (Proposition 1.7). In Section 2, we show that the number $\mathrm{Num}(G, n)$ is equal to the order of $\mathrm{Pic}_n(G)$ under certain hypotheses (Theorem 2.1). In particular, if G is a nilpotent group of odd order, then the number $\mathrm{Num}(G, n)$ is equal to the order of $\mathrm{Pic}_n(G)$ (Corollary 2.7). In Section 3, we compute the order of $\mathrm{Pic}_n(G)$ in general (Theorem 3.6). If a homotopy representation X has a G-homotopy type of a finite G-CW-complex, we call it *finite*. In Section 4, we discuss a similar problem for finite homotopy representations. However it seems difficult to compute the number $\mathrm{Num}_f(G, n)$ of G-homotopy types of finite homotopy representations with the same dimension function n because of complexity of the finiteness obstruction. When G is an abelian group of odd order, the number $\mathrm{Num}_f(G, n)$ is described by using the Swan homomorphisms