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Singular hyperbolic systems, VI. Asymptotic analysis for Fuchsian hyperbolic equations in Gevrey classes

By Hidetoshi TAHARA

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In the previous papers [**11**, **12**, **13**], the author has investigated Fuchsian hyperbolic equations in C^{∞} function spaces. But, here, Fuchsian hyperbolic equations are studied in Gevrey function spaces.

The motivation is as follows. Let

$$P=(t\partial_t)^2-t^{2\kappa_1}\partial_{x_1}^2-t^{2\kappa_2}\partial_{x_2}^2+t^{l_1}a_1(t, x)\partial_{x_1}+t^{l_2}a_2(t, x)\partial_{x_2}+b(t, x)(t\partial_t)+c(t, x),$$

where $(t, x)=(t, x_1, x_2)\in[0, T]\times \boldsymbol{R}^2$, $2\kappa_1, 2\kappa_2, l_1, l_2\in \boldsymbol{N}$ $(=\{1, 2, 3, \cdots\})$, $a_1(t, x)$, $a_2(t, x)$, $b(t, x)$, $c(t, x)\in C^{\infty}([0, T]\times \boldsymbol{R}^2)$, $a_1(0, x)\not\equiv 0$ and $a_2(0, x)\not\equiv 0$. Let $\rho_1(x)$, $\rho_2(x)$ be the roots of $\rho^2+b(0, x)\rho+c(0, x)=0$ and assume that $\rho_1(x), \rho_2(x)\notin \boldsymbol{Z}_+$ $(=\{0, 1, 2, \cdots\})$ for any $x\in \boldsymbol{R}^2$. Then, by Tahara [**11**] and Mandai [**7**] we can see the following: $Pu=f$ is well-posed in $C^{\infty}([0, T]\times \boldsymbol{R}^2)$, if and only if "$l_1\geqq\kappa_1$ and $l_2\geqq\kappa_2$" holds. Hence, if we want to treat P without "$l_1\geqq\kappa_1$ and $l_2\geqq\kappa_2$", we must restrict ourselves to the study in suitable subclasses of $C^{\infty}([0, T]\times \boldsymbol{R}^2)$. For this purpose, Gevrey classes seem to be very fitting. This is the reason why the author has come to treat the equation in Gevrey classes.

§1. Main Theorem.

First, we state our Main Theorem and its background.

Let $(t, x)\in[0, T]\times \boldsymbol{R}^n$ $(T>0)$, and let us consider

$$P(t, x, t\partial_t, \partial_x)=(t\partial_t)^m+\sum_{\substack{j+|\alpha|\leq m\\ j<m}} t^{l(j,\alpha)}a_{j,\alpha}(t, x)(t\partial_t)^j\partial_x^{\alpha}, \tag{1.1}$$

where $x=(x_1, \cdots, x_n)$, $\partial_t=\partial/\partial t$, $\partial_x=(\partial/\partial x_1, \cdots, \partial/\partial x_n)$, $m\in \boldsymbol{N}$ $(=\{1, 2, 3, \cdots\})$, $\alpha=(\alpha_1, \cdots, \alpha_n)\in \boldsymbol{Z}_+^n$ $(=\{0, 1, 2, \cdots\}^n)$, $|\alpha|=\alpha_1+\cdots+\alpha_n$ and $\partial_x^{\alpha}=(\partial/\partial x_1)^{\alpha_1}\cdots(\partial/\partial x_n)^{\alpha_n}$. Assume the following conditions:

(A_{κ}) $l(j, \alpha)\in \boldsymbol{R}$ $(j+|\alpha|\leqq m$ and $j<m)$ satisfy

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