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Structure of the scattering operator for time-periodic Schrödinger equations

By Shu NAKAMURA

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§1. Introduction.

In this paper we study the structure of the scattering operator for time-periodic Schrödinger equations with period ω:

$$i\frac{\partial}{\partial t}\phi(t, x)=(-\Delta+V(t, x))\phi(t, x), \qquad \phi(t, \cdot)\in\mathcal{H}=L^2(\boldsymbol{R}^n), \tag{1.1}$$

$$V(t+\omega, x)=V(t, x)\in\boldsymbol{R}, \qquad (t\in\boldsymbol{R},\ x\in\boldsymbol{R}^n). \tag{1.2}$$

Under suitable conditions on $V(t, x)$ to be specified below, (1.1) generates a unitary evolution operator $U(t, s)$, $-\infty<t, s<\infty$, and for each $s\in\boldsymbol{R}$, the wave operators defined by

$$W_{\pm}(s)=\underset{t\to\pm\infty}{\text{s-lim}}\, U(s, t)e^{-i(t-s)H_0}, \qquad H_0=-\Delta, \tag{1.3}$$

exist and are complete: $\operatorname{Ran} W_{\pm}(s)=\mathcal{H}^{ac}(U(s+\omega, s))$ (see Yajima [**14**], Howland [**6**], Kitada-Yajima [**8**], Nakamura [**11**]). Then the scattering operator defined by

$$S(s)=W_+(s)^*W_-(s) \tag{1.4}$$

is unitary, and by virtue of the time-periodicity, it satisfies

$$S(s)e^{-i\omega H_0}=e^{-i\omega H_0}S(s). \tag{1.5}$$

It follows that, if we denote a spectral representation of H_0 by $(\tilde{F}(\lambda), \mathcal{X}(\lambda), d\lambda)$, $S(s)$ is decomposed as

$$\begin{cases} S(s)=\sum_{\mu\in\boldsymbol{Z}} S_\mu, \\ \tilde{F}\left(\lambda-\frac{2\pi}{\omega}\mu\right)S_\mu\phi=\tilde{S}_\mu(\lambda)\tilde{F}(\lambda)\phi \quad (\text{a.e. } \lambda), \\ \tilde{S}_\mu(\lambda)\in \mathrm{B}\left(\mathcal{X}(\lambda), \mathcal{X}\left(\lambda-\frac{2\pi}{\omega}\mu\right)\right), \end{cases} \tag{1.6}$$

for any $\phi\in\mathcal{H}$. We call $\{\tilde{S}_\mu(\lambda)\}$ S-matrices (see §2 for details).

In this paper we are concerned with the structure of $S(s)$ or $\{\tilde{S}_\mu(\lambda)\}$, and show that the decay of $\tilde{S}_\mu(\lambda)$ as μ tends to infinity is completely determined by