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On a skew polynomial ring

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§ 0. Introduction.

If K is a commutative ring, and A is a commutative Frobenius K-algebra generated by a single element, then there exists a quasi-monic polynomial $f(X)$ such that $K[X]/(f(X))$ is K-algebra isomorphic to A, and conversely ([**6**]). In the preceding paper [**8**], as a generalization of this result, we have studied non-singular bilinear maps which come from a (*)-positively filtered ring $R=\bigcup_{i\geqq 0}R_i$ over a not necessarily commutative ring K. In the present paper we study the case when R_1/K is isomorphic to K as right K-module (or equivalently, as left K-module). In this case, $R=\bigcup_{i\geqq 0}R_i$ is a skew polynomial ring $K[X;\rho, D]=K\oplus XK\oplus X^2K\oplus\cdots$, which is defined by an automorphism ρ of K, a ρ-derivation D of K, and $aX=X\rho(a)+D(a)$ $(a\in K)$. Therefore some arguments proceed as in the case when R is a polynomial ring over a commutative ring, and more explicit descriptions may be obtained. Proposition 1.4 and Theorem 1.8 in this paper correspond to [**6**; Proposition 2.1], and Theorem 1.10 is analogous to [**6**; Proposition 3.2]. Further, Theorem 1.3 is a concrete description of [**8**; Theorem 12], and Theorem 1.9 corresponds to [**5**; Proposition 2.4]. However, the characterization of a separable polynomial (Theorem 1.8) is not enough. Because it is not easy to check that condition in practice. In fact, a monic polynomial $f(X)$ over a commutative ring is separable if and only if its discriminant (i.e. the resultant of $f(X)$ and $f'(X)$) is invertible, by virtue of the local criterion for a separable algebra. But, in our situation, we don't have a corresponding result. Consequently, in § 2 and § 3, we give several examples of separable polynomials. Further, concerning a separable polynomial, we have an open problem: Is a separable polynomial always Frobenius (i.e. its residue class ring is Frobenius over K)? Some arguments on this problem are done in § 3 (Theorem 3.4 and Theorem 3.5). It is easy to find a monic polynomial f such that $Rf=fR$ and R/Rf is not Frobenius over K. But we don't know as yet a separable polynomial which is not Frobenius. Finally, needless to say, the condition $Rf=fR$ for a monic polynomial f is not an easy one (Lemma 1.14 and Lemma 2.1). Concerning this, if K is an indecomposable commutative ring, every non-zero K-R-submodule I of R such that R/I is finitely generated and pro-