J. Math. Soc. Japan
Vol. 31, No. 2, 1979

Squeezing deformations in Schottky spaces

By Hiro-o YAMAMOTO

(Received March 24, 1977)
(Revised Aug. 7, 1978)

It is well known that any compact Riemann surface can be represented by a Schottky group. The so-called Schottky space, the set of all normalized marked Schottky groups of genus p, has been investigated by many authors. On the other hand, the concept of squeezing deformations of Riemann surfaces is powerful to treat boundaries of spaces of Kleinian groups.

If a compact Riemann surface S of genus p is squeezed with respect to a homotopically independent set $\{\alpha_i\}_{i=1}^{q}$ of loops on S, then there is a path in the Schottky space of genus p which tends to the boundary of the space. The aim of this paper is to study squeezing deformations of Riemann surfaces by investigating the behavior of the path in the Schottky space.

In § 1 we shall state the definition of squeezing deformations. Some useful properties in the later discussions are proved in § 2. In § 4 we define the Schottky space $\mathcal{S}_p$ of genus p and the boundary of $\mathcal{S}_p$, and classify the boundary points of $\mathcal{S}_p$ along the line due to Bers and Chuckrow. § 5 is devoted to prove the main Theorem 5.1 which asserts that, if a compact Riemann surface of genus p is squeezed with respect to a homotopically independent set $\{\alpha_i\}_{i=1}^{q}$ of A-cycles, then the path in $\mathcal{S}_p$ corresponding to the deformation tends to a cusp. We shall show another main Theorem 6.1 in § 6 which states that, if a compact Riemann surface of genus p is squeezed with respect to a homotopically independent set $\{\alpha_i\}_{i=1}^{q}$ of B-cycles, then the path in $\mathcal{S}_p$ corresponding to the deformation tends to a node.

The above theorems correspond to those in Abikoff [1] which concern with squeezing deformations in the Teichmüller space of a finitely generated Fuchsian groups of the first kind.

The author is indebted to Professors Oikawa, Kuroda and Taniguchi for their suggestions and continual encouragement during his preparation of this paper.

§ 1. The definition of squeezing deformations.

Let Γ be a finitely or an infinitely generated Kleinian group acting on the extended complex plane $\hat{C}$ and let $\Omega(\Gamma)$ be the region of discontinuity of Γ.