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Markov partitions of hyperbolic sets

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§ 1. Introduction and preliminaries.

In this paper we prove that pseudo-orbits near a hyperbolic set are shadowed, and for a hyperbolic set there is its arbitrarily slight extension which has Markov partitions.

Bowen proved similar results for a basic set of an Axiom A diffeomorphism ([1], [3]).

Suppose $f: M \to M$ is a diffeomorphism of a Riemannian manifold M with some Riemannian metric $\|\cdot\|$. A compact f-invariant set $\Lambda \subset M$ is said to be hyperbolic if $T_\Lambda M$ (the tangent bundle of M over Λ) splits into a Whitney sum of Tf-invariant subbundles

$$T_\Lambda M = E^s \oplus E^u,$$

and if there are $c>0$ and $0<\lambda<1$ such that

$$\|Tf^n v\| \leqq c\lambda^n \|v\| \qquad \text{if} \quad v \in E^s$$

$$\|Tf^{-n} v\| \leqq c\lambda^n \|v\| \qquad \text{if} \quad v \in E^u$$

for $n>0$.

The following was proved in [7].

THEOREM 0. *Suppose Λ is a hyperbolic set for a diffeomorphism $f: M \to M$, and U is a neighbourhood of Λ. Then there is a hyperbolic set Λ' with $\Lambda \subset \Lambda' \subset U$ which is a quotient of a subshift of finite type. More precisely there are a subshift of finite type Σ on symbols $\mathcal{B} = \{B_1, \cdots, B_N\}$ determined by an $N \times N$ 0-1 matrix $T = (t_{ij})$, and a map $\pi: \Sigma \to \Lambda'$ satisfying the followings.*

(1) *$\pi(\Sigma) = \Lambda'$ and $f\pi = \pi\sigma$, where σ is the shift transformation.*

(2) *B_i is a topologically embedded m-disk in M for $i=1, \cdots, N$.*

(3) *The map π is given by*

$$\pi((a_i)_{i \in \boldsymbol{Z}}) = \bigcap_{i \in \boldsymbol{Z}} f^{-i}(a_i) \qquad \textit{for} \quad (a_i)_{i \in \boldsymbol{Z}} \in \Sigma.$$

Here $\boldsymbol{Z}$ denotes the integers.