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# On the automorphism of $C^2$ with invariant axes

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## 0. Statement of results.

In this paper we study the biholomorphic automorphism of $C^2$ which leaves two coordinate axes invariant. E. Peschl investigated the automorphism of this type in [1]. We say such an automorphism is of axial type. If $F=(f(x, y), g(x, y))$ is an automorphism of axial type, then $F$ takes the form;

$$F:\begin{cases} f=xe^{\phi(x,y)} \\ g=ye^{\psi(x,y)}, \end{cases}$$

where $\phi$ and $\psi$ are holomorphic functions. We say that a function $f(x, y)$ is a component of an automorphism (of axial type) if there is a function $g(x, y)$ such that

$$T:\begin{cases} x'=f(x, y) \\ y'=g(x, y) \end{cases}$$

is an automorphism (of axial type).

Our results are as follows.

THEOREM. (1) *Let $\phi(x, y)$ be a polynomial and set $f(x, y)=xe^{\phi(x,y)}$. Then $f(x, y)$ is a component of an automorphism of axial type if and only if $\phi(x, y)$ takes the form $A(x^m y^{n+1})$, where $m$ and $n$ are non-negative integers and $A$ is a polynomial of one variable.*

(2) *The transformation*

$$T:\begin{cases} x'=xe^{A(x^m y^{n+1})} \\ y'=g(x, y) \end{cases}$$

*is an automorphism of axial type if and only if $g$ takes the form*

$$y\cdot\exp\left[-\frac{m}{n+1}A(x^m y^{n+1})+H(x')\right],$$

*where $H$ is a holomorphic function of one variable.*