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On mean ergodic theorems for positive operators in Lebesgue space

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§ 1. Introduction.

Let $(X, \mathcal{M}, m)$ be a σ-finite measure space and $L_p(X)=L_p(X, \mathcal{M}, m)$, $1 \leqq p \leqq \infty$, the usual (complex) Banach spaces. Let T be a bounded linear operator on $L_1(X)$ and τ its linear modulus [2]. In [9] (see also Akcoglu and Sucheston [1]) the author proved that if the adjoint of τ has a strictly positive subinvariant function in $L_\infty(X)$ then the following two conditions are quivalent: (i) T^n converges weakly; (ii) $\frac{1}{n}\sum_{i=1}^{n} T^{k_i}$ converges strongly for any strictly increasing sequence $k_1, k_2, \cdots$ of nonnegative integers. In the present paper we shall prove that if T is positive and satisfies $Tf=f$ whenever $0 \leqq f \in L_1(X)$ and $Tf \geqq f$, then the equivalence of (i) and (ii) still holds. Applying this result, we obtain that if, in addition, $\sup_n \|T^n\|_1 < \infty$ and if $T^n f$ converges weakly for any $f \in L_1(X)$ with $\int f\, dm=0$, then $\frac{1}{n}\sum_{i=1}^{n} T^{k_i} f$ converges strongly for any $f \in L_1(X)$ with $\int f\, dm=0$ and for any strictly increasing sequence $k_1, k_2, \cdots$ of nonnegative integers.

§ 2. Mean ergodic theorems.

In this section we shall assume that T is a *positive* linear operator on $L_1(X)$. T^* denotes the adjoint of T. Thus T^* acts on $L_\infty(X)$, and $\int (Tf)u\, dm = \int f(T^*u)\, dm$ for all $f \in L_1(X)$ and all $u \in L_\infty(X)$. If $A \in \mathcal{M}$ then 1_A is the indicator function of A and $L_p(A)$ denotes the Banach space of all $L_p(X)$-functions that vanish a. e. on $X-A$. A set $A \in \mathcal{M}$ is called *closed* under T if $f \in L_1(A)$ implies $Tf \in L_1(A)$.

The following proposition is stated with more generality than what is needed for applications in this paper. In particular, it extends a result of Lin [7, Theorem 1.1] (see also Krengel and Sucheston [5] and Lin [6]).