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The congruence monodromy problems

Dedicated to Professor Shôkichi Iyanaga on his 60th birthday

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Introduction

This is a summary of the forthcoming lecture note [**1**]. All details and proofs of the theorems will be given in [**1**], and are omitted here.

§ **0-1. The problems.** Let $G=\mathrm{PSL}_2(R)\times\mathrm{PSL}_2(k_{\mathfrak{p}})$, where R and $k_{\mathfrak{p}}$ are the real number field and a $\mathfrak{p}$-adic number field with $N\mathfrak{p}=q$ respectively, and $\mathrm{PSL}_2=\mathrm{SL}_2/\pm 1$. Let Γ be a torsion-free discrete subgroup of G with compact quotient, having a dense image of projection in each component of G. Our subject is such a discrete subgroup Γ. This study was motivated by the following series of conjectures which were suggested by our previous work [**2**]*. Since our group Γ is essentially nonabelian (see § 1-5, property (iv)), the readers will see that, by our conjectures, Γ would describe a "*non-abelian class field theory*" over an algebraic function field of one variable with finite constant field F_{q^2}. We would like to call the problems of determination of the validity of these conjectures, *the congruence monodromy problems*.

Conjectures**. With each Γ, we can associate an algebraic function field K of one variable with finite constant field F_{q^2} and with genus $g\geqq 2$, and a finite set $\mathfrak{S}(K)$ consisting of $(q-1)(g-1)$ prime divisors of K of degree one over F_{q^2}, satisfying the following properties. Here, the elements of $\mathfrak{S}(K)$ are called *the exceptional prime divisors*, while all other prime divisors of K are called *the ordinary prime divisors*.

CONJECTURE 1. *The ordinary prime divisors P of K are in one-to-one correspondence with the pairs $\{\gamma_P^{\pm 1}\}_\Gamma$ of mutually inverse primitive elliptic conjugacy classes of Γ* (see § 1 for the definitions).

CONJECTURE 2. *The finite unramified extensions K' of K, in which all $(q-1)(g-1)$ exceptional prime divisors of K are decomposed completely, are in*

* The proofs of results stated in [**2**] will also be given in [**1**]. There is some overlap between a part of § 2 of [**2**] and § 1 of this paper.

** See also § 3.