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On normal contact metric manifolds

by Minoru KURITA

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In a series of papers [3], [4], [5], [6] S. Sasaki and his collaborators studied what they called an almost contact structure on an odd-dimensional manifold, which could be regarded as a structure corresponding to an almost complex one on an even-dimensional manifold, and was called so because of its close connection with a contact structure defined by a 1-form of maximal rank.

In the first part of this paper we treat Sasaki's theory by the method of adapted frames and in the second we investigate fundamental properties of a normal contact metric structure defined also by Sasaki and closely related to a Kaehlerian structure.

DEFINITIONS. Throughout the paper we assume manifolds and tensors to be real analytic, because we discuss a complete integrability of differential equations in complex domain. An almost contact structure, or (ϕ, ξ, η)-structure, on a $2n+1$-dimensional differentiable manifold M with local coordinates $x^1, \cdots, x^{2n+1}$ is defined by a tensor field $\phi=(\phi_i^j)$ and two vector fields $\xi=(\xi^1, \cdots, \xi^{2n+1})$, $\eta=(\eta_1, \cdots, \eta_{2n+1})$ such that

$$\begin{gathered} \operatorname{rank}\phi=2n, \qquad \phi_j^i\xi^j=0, \qquad \phi_i^j\eta_j=0, \qquad \xi^i\eta_i=1, \\ \phi_i^j\phi_j^k=-\delta_i^k+\xi^k\eta_i \qquad (i, j, k=1, \cdots, 2n+1). \end{gathered} \tag{1}$$

When we consider ϕ as a matrix with an element ϕ_i^j on the i-th row and the j-th column, we have

$$\phi^3+\phi=0. \tag{2}$$

We take a one dimensional space R with a coordinate t $(-\infty<t<+\infty)$ and construct an almost complex tensor $F=(f_A^B)$ on $M\times R$ as follows

$$f_i^j=\phi_i^j, \quad f_{2n+2}^i=\xi^i, \quad f_i^{2n+2}=-\eta_i, \quad f_{2n+2}^{2n+2}=0. \tag{3}$$

The manifold M is called *normal* when the Nijenhuis tensor N of the tensor F vanishes.

An almost contact metric structure, or (ϕ, ξ, η, g)-structure, is defined as an almost contact structure with a Riemannian metric $g=(g_{ij})$ such that

$$g_{ij}\xi^j=\eta_i, \quad g_{ij}\phi_h^i\phi_k^j=g_{hk}-\eta_h\eta_k. \tag{4}$$

A contact metric structure is defined as follows. On an almost contact