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# On Artin's L-functions of the algebraic curves uniformized by certain automorphic functions

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## Introduction.

Recently, S. S. Rangachari [4], considered Artin's $L$-functions as an analogy of Hasse's zeta-function, for the algebraic curves uniformized by modular functions belonging to congruence subgroups. He proved, in certain cases, that Artin's $L$-function can be expressed as a product of Dirichlet series, corresponding to cusp forms of degree 2, obtained by Hecke [3]. On the other hand G. Shimura [7] proved that Hasse's conjecture is also true for algebraic curves uniformized by automorphic functions belonging to an indefinite quaternion algebra. Argebraic curves of this type include the curves treated by Rangachari as special case. In this paper, we consider Artin's $L$-functions for those curves.

Our principal result is as follows. Let $\Phi$ be an indefinite quaternion algebra over the rational number field $\boldsymbol{Q}$, $\mathfrak{o}$ be a maximal order in $\Phi$. For a positive integer $N$ prime to the discriminant $d(\Phi)$ of $\Phi$, we denote by $\Gamma$ and $\Gamma_N$, respectively, the group of units in $\mathfrak{o}$ with positive reduced norm and its subgroup consisting of elements such that $\gamma\equiv 1 \bmod N$. As $\Phi$ has a faithfull representation by real matrices of degree 2, $\Gamma$ and $\Gamma_N$ are considered as Fuchsian groups on the upper half plane $\mathfrak{H}$. We can find an algebraic curve $\mathfrak{L}_N$ defined over $\boldsymbol{Q}$, whose function field is the field of automorphic functions with respect to the group $\Gamma_N$. Let $\rho_1$ be an absolutely irreducible representation of the group $\Gamma/\Gamma_N=G$ contained in the analytic representation of $G$ on the jacobian variety of $\mathfrak{L}_N$. Considering $G$ as a subgroup of the group $\Omega=GL(2,\boldsymbol{Z}/N\boldsymbol{Z})/\{\pm 1\}$, we denote by $\{\rho_1,\cdots,\rho_r\}$ the set of all inequivalent conjugate representations of $\rho_1$ relative to $\Omega$. Put $\deg\rho_1=m$ and $\chi=\chi_1+\cdots+\chi_r$ with $\chi_i=\operatorname{tr}\rho_i$. Then our main theorem asserts that the $m$-th power of Artin's $L$-function $L(\chi,s)^m$ defined for the curves can be expressed as a product of Dirichlet series obtained from the representation of modular correspondences by automorphic cusp forms of type $(\Gamma,\rho,2)$ which are considered in [8]. Therefore, such a power $L(\chi,s)^m$ is meromorphic on the whole $s$-plane and satisfies a functional equation (Theorem 1). Further we prove that, if the