J. Math. Soc. Japan
Vol. 14, No. 2, 1962

# Ordered idempotent semigroups

By Tôru SAITÔ

(Received March 13, 1961)
(Revised Sept. 25, 1961)

**1.** By an *ordered semigroup*, we mean a system $S(\cdot, <)$, which satisfies the following conditions:

I. $S$ is a semigroup with respect to the multiplication $\cdot$;

II. $S$ is simply ordered by $<$;

III. if $a$ and $b$ are elements of $S$ such that $a < b$, then $ac \leqq bc$ and $ca \leqq cb$ for all $c \in S$.

Many authors, especially Alimov [**1**], Clifford [**2**], [**4**], [**5**], Hion [**8**] and Conrad [**6**], studied such semigroups with some restrictions. Alimov studied ordered semigroups which satisfy the conditions I, II and the stronger

III$'$. if $a$ and $b$ are elements of $S$ such that $a < b$, then $ac < bc$ and $ca < cb$ for all $c \in S$.

But many ordered topological semigroups do not satisfy the condition III$'$. The remaining authors made rather artificial restrictions, and, as far as we know, none discussed ordered semigroups in our general sense. Indeed, the structure of such semigroups seems to be very complicated. However, it can be seen that, in any ordered semigroup, the set of all idempotents, if it is non-void, constitutes a subsemigroup. And, in this note, as the first step of the general study of ordered semigroups, we treat ordered idempotent semigroups i.e. ordered semigroups in which every element is idempotent.

In §§ **2–6** we discuss the structure of ordered idempotent semigroups, and in § **7** we show that some properties in previous sections, characterize ordered idempotent semigroups. As an appendix of this note, in the final § **8**, we remark the characterizations of two special idempotent semigroups.

**2.** In this section, we give some preliminary properties of ordered idempotent semigroups. First we mention, without proof, an interesting following result of Clifford about idempotet semigroups.

LEMMA 1 (Theorem 3 of Clifford [**3**]). *An idempotent semigroup $S$ can be decomposed into subsets $\{D_\alpha : \alpha \in A\}$ in such a way that*

(i) *for any pair of subsets $D_\alpha$ and $D_\beta$, the set $\{xy, yx ; x \in D_\alpha, y \in D_\beta\}$ is contained in a third $D_\gamma$, and that*

(ii) *all the elements of every $D_\alpha$ can be arranged in a rectangular form, such*