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Multiple Wiener Integral

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The notion of *multiple Wiener integral* was introduced first by N. Wiener[1] who termed it *polynomial chaos*. Our definition in the present paper is obtained by a slight modification of Wiener's one, and seems to be more convenient in the point that our integrals of different degrees are orthogonal to each other while Wiener's polynomial chaos has not such a property.

In § 1 we shall define a normal random measure as a generalization of a brownian motion process. In § 2 we shall define multiple Wiener integral and show its fundamental property. In § 3 we shall establish a close relation between our integrals and Hermite polynomials. By making use of this relation we shall give, in § 4, an orthogonal expansion of any L_2-functional of the normal random measure, which proves to be coincident with the expansion given by S. Kakutani[2] for the purpose of the spectral resolution of the shift operator in the L_2 over the brownian motion process. In § 5 we shall treat the case of a brownian motion process, and in this case we shall show that we can define the multiple Wiener integral by the iteration of stochastic integrals.[3]

§ 1. Normal random measure

A system of real random veriables $\xi_\alpha(\omega)$, $\alpha \in A$, ω being a probability parameter, is called normal when the joint destribution of $\xi_{\alpha_1}, \dots, \xi_{\alpha_n}$; $\alpha_1, \dots, \alpha_n \in A$, is always a multivariate Gaussian distribution (including degenerate cases) with the mean vector $(0, \cdots, 0)$.

By making use of Kolmogoroff's theorem[4] of introducing a probability distribution in R^A, we can easily prove the following

1) N. Wiener: The homogeneous chaos, Amer. Journ. Math. Vol. **LV**, No. 4, 1938.

2) S. Kakutani: Determination of the spectrum of the flow of Brownian motion, Proc. Nat. Acad. Sci., U.S.A. **36** (1950), 319–323.

3) K. Itô: Stochastic integral, Proc. Imp. Acad. Tokyo, Vol. **XX**, No. 8, 1944.

4) A. Kolmogoroff: Grundbegriffe der Wahrscheinlichkeitsrechnung, Berlin, 1933. The consistency-condition is well satisfied by virtue of the property of multivariate Gaussian distribution.