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On the jump-diffusion approximation of stochastic difference equations driven by a mixing sequence

By Tsukasa FUJIWARA

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§1. Introduction.

In this paper, we will establish a limit theorem for a sequence of stochastic processes determined by random difference equations. A feature of our work is that the limiting process is a Markov process with jumps whereas the pre-limiting processes are non-Markovian.

There has been extensive works concerning the problem of approximating non-Markovian process by diffusion. See, for example, Kesten and Papanicolau [**5**] and Khas'minskii [**6**]. Our aim is to extend these works to jump-diffusions (strong Markov processes with jumps) and it will give us some new aspects of limit theorems. In particular, we are interested in the problem how the continuous part and the jump part of limiting process come out. Recently, such extension has been studied by several authors. For example, Jacod and Shiryaev [**4**] gives a comprehensive survey of limit theorems in which they treat the weak convergence of semimartingales. Their standpoint of view is that the convergence of characteristics of semimartingales implies that of semimartingales, and their results include very general limit theorems. But we emphasize that the result of this paper is not contained in theirs, because our setting of problem is concerned with some mixing property which does not yield the convergence of characteristics in the sense of [**4**]. See Remark 3 in §2.

Now, the problem we will discuss is formulated as follows. For an $\boldsymbol{R}^e$-valued array $\{\xi_{n,k}\,;\, n, k\in\boldsymbol{N}\}$ of random variables and a $d\times e$-matrix valued function C on $\boldsymbol{R}^d$, we consider the stochastic difference equation:

$$(1.1)\qquad \begin{cases} \varphi_{n,k}-\varphi_{n,k-1}=C(\varphi_{n,k-1})(\xi_{n,k}-a_n)\,, & k=1,2,\cdots,\\ \varphi_{n,0}=x_0\in\boldsymbol{R}^d\,, \end{cases}$$

where we set $a_n=E[\xi_{n,1}I_{(0,1]}(|\xi_{n,1}|)]$ and I_A denotes the indicator function of a set A.

Let $\{j_n\}_n$ be a positive sequence diverging to infinity. Define a sequence of stochastic processes $\{\varphi_n\}_n$ by

$$(1.2)\qquad \varphi_n(t)=\varphi_{n,[j_n t]}$$