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Asymptotic behavior of some oscillatory integrals

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The purpose of the present note is to prove an asymptotic expansion theorem for a certain kind of oscillatory integrals. Our theorem is a generalization of Theorem 3.2.4 of Hörmander [1] in the sense that we allow the phase function to contain certain inhomogeneous terms. Our motivation which leads us to considering such a kind of generalization lies in long-range scattering (cf. [3], [4]) as against Hörmander's purpose in [1] was to consider Fourier integral operators. Using our main result, we can give a proof of Theorem 3.1 of [4] which played a crucial role in the proof of the completeness of the modified wave operator for long-range scattering. We should note that Hörmander's Theorem 3.2.4 also can be used to prove the completeness when long-range potential V satisfies $V(x)=O(|x|^{-1/2-\varepsilon})$, $\varepsilon>0$, but for V which has longer range, we must use our generalized theorem proved in this paper.

The crucial tool we shall use is the method of stationary phase (see e.g. Hörmander [1] and [2]). Moreover, a method similar to Hörmander's proof of Theorem 3.2.4 will be used to estimate the integral on the region bounded away from the critical point of the phase function.

We shall first summarize our main result in §1 and then prove it in §2.

§1. Main result.

We shall consider the distribution $A_{\omega,\varepsilon}$ defined by

(1.1) $$\langle A_{\omega,\varepsilon}, u\rangle = \int_{R^n}\int_{R^N} e^{i(\varphi(\omega;x,\theta)-X(x,\theta))}a(x,\theta)u(x)\chi(\varepsilon\theta)\,d\theta dx,$$

$$\varepsilon\neq 0,\quad u\in C_0^\infty(R^n),$$

where ω is a parameter; functions φ, X, a are C^∞; and χ is a rapidly decreasing function on R^N with $\chi(0)=1$. The precise conditions imposed on those functions will be given below. Under those conditions, we shall prove the