ON A WEAKLY CENTRAL OPERATOR ALGEBRA

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In the previous paper [7], we have defined the weak centrality of an operator algebra modifying I. Kaplansky's definition of the (strong) centrality [6]. Although we have assumed an additional condition in the previous occasion, here we shall study the weak centrality as itself. It will be seen in the below, that a weak central C^*-algebra can be decomposed into C^*-algebras each of which is factorial C^*-algebra. The other purpose of this paper is to prove that any W^*-algebra is weakly central.

1. Definitions and notations. We shall assume in this papar that any algebra which we consider has a unit element. A self-adjoint algebra A of bounded linear operators on a Hilbert space will be called a *C^*-algebra* (*W^*-algebra*), according to I. E. Segal, provided that A is uniformly (weakly) closed in the sense of J. von Neumann [9].

Let Ω be the set of all maximal ideals in the C^*-algebra A. In simple case we shall consider the 0-ideal as its maximal ideal. If S is a nonvacuous subset of Ω, we define M_0 is contained in the closure of S if and only if $M_0 \supset \bigcap_{M \in S} M$. This topological space Ω will be called the *spectrum* of A, according to I. E. Segal [10]. The spectrum Ω becomes, in general, a compact T_1-space. In the commutative case, it is known that the spectrum becomes a T_2-space.

A C^*-algebra A is called *weakly central* provided that two maximal ideals M_1 and M_2 coincide if and only if

$$M_1 \cap Z = M_2 \cap Z$$

where Z means the center of A. It will be seen that A is weakly central if it has at most one maximal ideal. Conversely, if the center of a weakly central algebra A is a field, then A contains at most one maximal ideal. It is not difficult to see that the spectrum of a weakly central C^*-algebra is homeomorphic in its natural mapping to the spectrum of the center, whence it is a compact T_2-space.

In the terminology of N. Jacobson [4], an ideal P of A is called *primitive* if there exists a maximal right ideal M' such that

$$M' : A = \{x \in A \mid ax \in M' \text{ for all } a \in A\} = P.$$

It is known that a C^*-algebra is *semi-simple* in the sense of Jacobson [4], i. e., the intersection of all primitive ideals in a C^*-algebra vanishes. The set of all primitive ideals is called the *structure space* of the algebra with the Stone topogy. A C^*-algebra is *central* if and only if the definitive property of the weak centrality is held for primitive ideals in stead of maximals ideals.