ON COVERING SURFACES OF A CLOSED RIEMANN SURFACE OF GENUS $p \geqq 2$

MASATSUGU TSUJI

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1. Let F be a closed Riemann surface of genus $p \geqq 2$ spread over the z-plane. We cut F along p disjoint ring cuts C_i $(i = 1, 2, \cdots, p)$ and let F_0 be the resulting surface. We take infinitely many same samples as F_0 and connect them along the opposite shores of C_i as in the well known way, then we obtain a covering surface $F^{(\infty)}$ of F, which is of planar character. Hence by Koebe's theorem, we can map $F^{(\infty)}$ conformally on a schlicht domain D on the ζ-plane, whose boundary E is a non-dense perfect set, which is the singular set of a certain linear group of Schottky type. Myrberg[1] proved:

THEOREM 1. *E is of positive logarithmic capacity.*

In another paper[2], I have proved:

THEOREM 2. *Every point of E is a regular point for Dirichlet problem.*

Hence $F^{(\infty)}$ is of positive boundary and its Green's function $G(z, z_0)$ tends to zero, when z tends to the ideal boundary of $F^{(\infty)}$. Now instead of cutting F along p ring cuts, we cut F along q $(1 \leqq q \leqq p)$ ring cuts C_i $(i = 1, 2, \cdots, q)$ and let F_0 be the resulting surface. We take infinitely many same samples as F_0 and connect them along the opposite shores of C_i $(i = 1, 2, \cdots, q)$, then we obtain a covering surface $F^{(\infty)}_{(q)}$ of F. Then I have proved in another paper[3] the following extension of Theorem 1.

THEOREM 3. *$F^{(\infty)}_{(1)}$ is of null boundary, while if $q \geqq 2$, $F^{(\infty)}_{(q)}$ is of positive boundary and there exists a non-constant bounded harmonic funtion on $F^{(\infty)}_{(q)}$, whose Dirichlet integral is finite.*

In this paper, we shall prove the following extension of Theorem 2.

THEOREM 4. *The Green's function $G(z, z_0)$ of $F^{(\infty)}_{(q)}$ $(q \geqq 2)$ tends to zero, when z tends to the ideal boundary of $F^{(\infty)}_{(q)}$.*

2. Let Φ be a non-compact subsurface of $F^{(\infty)}_{(q)}$ $(q \geqq 2)$ whose compact boundary

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