ON THE GENERALIZED INVARIANT DIFFERENTIAL FORM ON MANIFOLDS WITH GENERAL CONNECTION

HIROYOSHI SASAYAMA

(Received July 6, 1953)

0. Introduction. It was shown by E. Cartan [1][1)] that any invariant differential form in an n-dimensional homogeneous space with a finite, continuous transformation group of Lie as the group of structure is a certain exterior form with constant coefficients invariant under the linear group of isotropy of the space and conversely. Recently, his famous two theorems for compact isogeneous spaces, except the above one in his paper, that any closed differential form is equivalent to an invariant differential form and that any invariant differential form equivalent to 0 is invariantly null are generalized for connected manifolds of class C^2 with a compact connected topological group of transformations by C. Chevalley and S. Eilenberg [2]. On the other hand, late H. Iwamoto studied differential forms invariant under the holonomy group in compact, orientable, positive-definite Riemannian spaces [3], which form a contrast in certain sense with invariant differential forms in compact isogeneous spaces especially when both spaces are symmetric. A trial in the following is to generalize the above E. Cartan's theorem first stated for abstract manifolds with general connection which may be nonlinear in the style of C. Chevalley and moreover so as to include the generalization for homogeneous spaces with a topological transtormation group as the group of structure as a special case. Since, exterior differential forms in Banach spaces were treated by M. Kerner [4], we shall take general manifolds with Banach coordinate in our consideration.

It will be seen that spaces with projective connection and spaces with conformal connection must be excluded and that if tangent spaces be nonlinear, the condition that the points of contact are changed to another points of contact under displacements must be imposed.

1. The general homogeneous holonomy group. Let *a manifold X of class $C^k (k \geqq 2)$ with Banach coordinate, general connection and a topological transformation group G as the fundamental group* be defined as follows:

(1.1) X is *a general manifold of class C^k with Banach space C as the local coordinate space*, namely, a connected topological space with neighborhoods U_p at each point p such that to each point p, a class $\mathfrak{F}_p = \{f\}$ of functionals $f: X \to C$, called the functionals of class C^k on X at p, defined in a

1) Numbers in brackets refer to the references at the end of the paper.