CONFORMAL MAPS OF ALMOST KAEHLERIAN MANIFOLDS

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1. Consider a campact Riemannian manifold M[2] and let Γ be a connection in the bundle of orthogonal frames over M whose homogeneous holonomy group is irreducible. Then, with respect to Γ every affine transformation is homothetic and hence an affine transformation with respect to the Riemannian connection γ. Since the largest connected group of affine transformations $A_0^\gamma(M)$ with respect to γ coincides with the largest connected group $I_0(M)$ of isometries [3] it follows that the largest connected group $A_0^\Gamma(M)$ of affine transformations with respect to Γ consists of isometries. In particular, if M is an almost Kaehlerian manifold, $A_0^\Gamma(M)$ coincides with the largest connected group of automorphisms of the almost Kaehlerian structure provided the homogeneous holonomy group associated with Γ is irreducible. Now, an infinitesimal conformal transformation of a compact flat space is homothetic (hence isometric), and so it is an infinitesimal automorphism of the almost Kaehlerian structure. On the other hand, if the Ricci scalar curvature is a non-positive constant an infinitesimal conformal transformation is isometric and the same conclusion prevails.

For compact Kaehlerian manifolds M it is known that the largest connected Lie group $C_0(M)$ of conformal transformations coincides with the largest connected group $\widetilde{A}_0(M)$ of automorphisms of the Kaehlerian structure provided the topological dimension of M is greater than 2. If dim $M = 2$ it coincides with the largest connected group of analytic homeomorphisms [6]. The following theorem was recently proved [4]:

If M^{4k} is a $4k$-dimensional compact almost Kaehlerian manifold then $C_0(M^{4k}) = \widetilde{A}_0(M^{4k})$.

It is one of the main purposes of this paper to extend this result so that it holds for all dimensions. We shall prove, in particular,

THEOREM 1. *The largest connected Lie group of conformal transformations of a compact almost Kaehlerian manifold $M^{2n} (n > 1)$ coincides with the largest connected group of automorphisms of the almost Kaehlerian structure, that is $C_0(M^{2n}) = \widetilde{A}_0(M^{2n})$.*

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2) The manifolds, differential forms and tensorfields considered are assumed to be of class C^∞.