CONVEXITY THEOREMS FOR ALLIED FOURIER SERIES

KENJI YANO

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Recently we proved a number of convexity theorems for Fourier series [2], and they were slightly generalized in [3]. The present paper is a continuation of [2, 3]. Indeed, we shall treat the allied Fourier series analogues.

With respect to Theorems 1, 2 and 6 in [2], the results for allied series will be different a little, while with respect to Theorems 3, 4 and 5 in [2], the results will be almost similar. For the sake of contrast, we shall number the theorems of this paper with the order of theorems in [2].

1. Notations. Let $\psi(t)$ be an odd function integrable in Lebesgue sense in $(0, \pi)$, and periodic of period 2π, and let

$$\psi(t) \sim \sum_{n=1}^{\infty} b_n \sin nt,$$

$$\bar{s}_n^r = \sum_{\nu=1}^{n} A_{n-\nu}^r b_\nu \qquad (-\infty < r < \infty),$$

$$\bar{t}_n^{r+1} = \sum_{\nu=1}^{n} A_{n-\nu}^r (\nu b_\nu) \qquad (-\infty < r < \infty),$$

$$\bar{\sigma}_n^r = \bar{s}_n^r / A_n^r \qquad (r > -1),$$

where $A_n^r = \binom{r+n}{n}$, $n = 0, 1, 2, \dots$. We write

$$\Psi_0(t) = \psi_0(t) = \psi(t),$$

$$\Psi_\beta(t) = \frac{1}{\Gamma(\beta)} \int_0^t (t-u)^{\beta-1} \psi(u)\, du \ (\beta > 0),$$

$$\psi_\beta(t) = \Gamma(\beta+1) t^{-\beta} \Psi_\beta(t) \ (\beta > 0).$$

Similarly, from the function

$$\theta(t) = \frac{2}{\pi} \int_t^{\infty} \frac{\psi(u)}{u}\, du$$

we define $\theta_\beta(t)$ for $\beta \geqq 0$. For the negative value of β, let

$$\theta_\beta(t) = t^{-\beta} \frac{d}{dt} \int_0^t (t-u)^\beta \theta(u)\, du \qquad (-1 < \beta < 0).$$