ON THE DECOMPOSITION THEOREMS OF FOURIER TRANSFORMS WITH WEIGHTED NORMS

SATORU IGARI

(Received October 23, 1962)

1. Introduction. Littlewood and Paley [6] proved the following result;

For $f(x) \in L^p(-\pi,\pi)\,(1<p<\infty)$, *let*

$$\hat{f}(\nu) = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{-i\nu x}dx, \tag{1. 1}$$

$$f(x) \sim \sum_{\nu=-\infty}^{\infty} \hat{f}(\nu)e^{i\nu x}. \tag{1. 2}$$

If

$$\Delta_n(x) = \begin{cases} \sum_{\nu=2^{n-1}}^{2^n-1} \hat{f}(\nu)e^{i\nu x} & n=1,\ 2,\cdots \\ \hat{f}(0) & n=0 \\ \sum_{\nu=-2^{-n}+1}^{-2^{-n-1}} \hat{f}(\nu)e^{i\nu x} & n=-1,\ 2,\cdots, \end{cases} \tag{1. 3}$$

then

$$0 < A_p \leqq \int_{-\pi}^{\pi}\left\{\sum_{n=-\infty}^{\infty}|\Delta_n(x)|^2\right\}^{p/2}dx \Big/ \int_{-\pi}^{\pi}|f(x)|^p dx \leqq A_p' < \infty.$$

Concerning this theorem, discrete and integral analogues were proved by G.Sunouchi [12], [11] and recently J.Schwartz [8] gave a new proof. On the other hand, the theorem just cited was extended by I.I.Hirschman Jr. to the weighted L^p-class (Theorems 6 and 7) and the Fourier integral case with the weighted norms was investigated by D.L.Guy [2] (Theorems 1 and 2). However their proofs are complicated.

In the present note we shall prove the integral, discrete and ordinary cases with weighted norms with the idea of J.Schwartz. Our main methods depend upon the extended Marcinkiewicz interpolation theorem and the test for an operator to be weak type $(1,q)$ due to L. Hörmander [5] which are applied to vector-valued functions by J.Schwartz [8], and another tool is of a substitute of Parseval's relation.

§§2-6 and §§7-8 are devoted to the proof of Fourier integral and discrete