APPROXIMATION AND SATURATION OF FUNCTIONS BY ARITHMETIC MEANS OF TRIGONOMETRIC INTERPOLATING POLYNOMIALS

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Let $f(x)$ be a continuous function with period 2π and E_n be the set of equidistant nodal points situated in the interval $0 \leqq x < 2\pi$, that is

$$\xi_0 + 2\pi j/(2n+1) \qquad (j = 0, 1, \cdots, 2n), \qquad (\mathrm{mod.}\ 2\pi)$$

where ξ_0 is any real number. Then the trigonometric polynomial of order n coinciding with $f(x)$ on E_n is

$$(1) \qquad I_n(x, f) = \frac{1}{\pi}\int_0^{2\pi} f(t) D_n(x-t)\, d\omega_{2n+1}(t),$$

where $D_n(x)$ is the Dirichlet kernel and $\omega_{2n+1}(t)$ is a step function which is associated with E_n. (We shall refer to A. Zygmund [4, Chap. X] these notations and fundamental properties of trigonometric interpolation.) We denote the Fourier expansions of (1) by

$$(2) \qquad I_n(x, f) = \sum_{k=-n}^{n} c_k^{(n)} e^{ikx}$$

$$c_k^{(n)} = \frac{1}{2\pi}\int_0^{2\pi} f(t) e^{-ikt} d\omega_{2n+1}(t).$$

The $\{c_k^{(n)}\}$ are called the k-th Fourier-Lagrange coefficients and for a fixed k, $c_k^{(n)}$ is an approximate Riemann sum for the integral defining Fourier coefficient c_k of $f(x)$, that is

$$c_k = \frac{1}{2\pi}\int_0^{2\pi} f(t) e^{-ikt} dt.$$

Let us denote the partial sums of (1) by

$$I_{n,m}(x, f) = \sum_{k=-m}^{m} c_k^{(n)} e^{ikx} \qquad (m \leqq n),$$

in particular

$$I_n(x, f) = I_{n,n}(x, f).$$